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Symmetrically generated groups

Benny Nguyen

SYMMETRICALLY GENERATED GROUPS

A Thesis

Presented to the

Faculty of

California State University

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In Partial Fulfillment

of the Requirements for the Degree

Masters of Arts

in

Mathematics

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Approved by:

[redacted] 6/8/05

Zahid Hasan, Committee Chair — Date

[redacted]

John Sarli, Committee Member

[redacted]

Peter Williams, Committee Member

[redacted]

Peter Williams, Chair
Department of Mathematics

[redacted]

Joan Terry Hallett,
Graduate Coordinator
Department of
Mathematics

ABSTRACT

In this thesis, we construct several groups entirely by hand via their symmetric presentations. In particular, we use the technique of double coset enumeration to manually construct $J_3:2$, the automorphism group of the Janko group J_3, and represent every element of the group as a permutation of $PSL_2(16):4$, on 120 letters, followed by a word of length at most 3.

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CHAPTER ONE

INTRODUCTION

Definitions

Symmetric generation methods provide a uniform way of constructing finite group. Since all finite groups can be constructed from simple groups, we are particularly interested in simple groups. Now every nonabelian simple group is a homomorphic image of 2^{*n}: N, a semi-direct product of n copies of a cyclic group of order 2 extended by a subgroup N of the symmetric group of degree n. Therefore, we investigate homomorphic images of 2^{*n}: N.

The methods of symmetric generation can be used to prove the existence of a finite group, and the main purpose of this thesis is to demonstrate how. We will use the methods and demonstrate how $PSL_3(2)$ occurs as the homomorphic image of 2^{*7}: $PSL_3(2)$.

The symbol m^{*n} denotes a free product of n copies of the cyclic group C_m; thus, $F = 2^{*n} = \langle t_1, t_2, ..., t_n \mid t_i^2 = 1 \rangle \cong \underbrace{C_2 * C_2 * ... * C_2}_{n-times}$.

If π is a permutation of the n letters {1, 2,..., n} then π induces an (outer) automorphism $\hat{\pi}$ of F which simply permutes the subscripts. Thus,

$$\hat{\pi} : t_i \to t_{i^\pi}.$$

In this way, if N is a permutation group on $\Lambda = \{1, 2, \ldots, n\}$, then we may embed N in Aut(F), and so form the semi-direct product

$F : N = 2^{*n} : N = \{\pi\omega \mid \text{where } \pi$ is an element of N and ω is a word of the $t_i\}$.

The product of two elements in 2^{*n}: N given by

$$\sigma u . \pi \upsilon = \sigma\pi u^{\pi} \upsilon \text{ ,}$$

where $\sigma, \pi \in N$, u and υ are words in the t_i, and the word $u^{\pi}\upsilon$ is put into canonical form by canceling any adjacent repetitions of the same t_i. When N is a transitive subgroup of the symmetric group S_n, $P = 2^{*n} : N$ is called a progenitor and N a control subgroup and the n involution t_i the symmetric generators (see Curtis[4]).

Any homomorphic image of P is obtained by mapping certain relators of the form $\pi\omega$ to the identity, and so the general image of P takes the form

$$G = \frac{2^{*n} : N}{\pi_1\omega_1, \pi_2\omega_2, \ldots}.$$

In other words we factor out the smallest normal subgroup of P containing the set $\{\pi_1\omega_1, \pi_2\omega_2, ..\}$. If the symmetric generators generate G, we say that G is a symmetrically generated group. In the cases that interest us the image of N will be isomorphic to N and the images of the t_i will have order 2 and be distinct.

Double coset decomposition:

Let G be a group defined by

$$G = \frac{2^{*n} : N}{\pi_1\omega_1, \pi_2\omega_2, \ldots}.$$

The coset stabilizing subgroup, $N^{(\omega)}$, of N is given by,

$$N^{(\omega)} = \{\pi \in N : N\omega\pi = N\omega\},$$

for ω a word in the symmetric generators.

Clearly $N^{\omega} \leq N^{(\omega)}$, and the number of cosets in the double coset $[\omega] = N\omega N$ is given by $|N|/|N^{(\omega)}|$, since

$$N\omega\pi_1 \neq N\omega\pi_2 \Leftrightarrow N\omega\pi_1\pi_2^{-1} \neq N\omega$$

$$\Leftrightarrow \pi_1\pi_2^{-1} \notin N^{(\omega)}$$

$$\Leftrightarrow N^{(\omega)}\pi_1\pi_2^{-1} \neq N^{(\omega)}$$

$$\Leftrightarrow N^{(\omega)}\pi_1 \neq N^{(\omega)}\pi_2$$

In order to obtain the index of N in G we shall perform a manual double coset enumeration of G over N; thus we must find all double cosets $[\omega]$ and work out how many single cosets each of them contains. We shall know that we have completed the double coset enumeration when the set of right cosets obtained is closed under right multiplication. Moreover, the completion test above is best performed by obtaining the orbits of $N^{(\omega)}$ on the symmetric generators. We need only identify, for each $[\omega]$, the double coset to which $N\omega t_i$ belongs for one symmetric generator t_i from each orbit.

We wish to decompose G into double cosets of the form NxN; that is to say we wish to find a set $\{x_1, x_2, \ldots\}$ of elements of G such that

$$G = Nx_1N \cup Nx_2N \cup \ldots$$

But for each i we have $x_i = \pi_i\omega_i$ for some $\pi_i \in N$ and some word ω_i in the t_i, and so the double coset decomposition simplifies to

$$G = N \cup N\omega_2 N \cup N\omega_3 N \cup \ \ldots,$$

where ω_1 is chosen to be the identity. When the set of relations by which we are factoring is empty this gives the double coset decomposition of the progenitor P, and we see that the double cosets simply correspond to the orbits of N on the ordered k-tuples of letters of Λ which have no adjacent repetitions, where $k \in N = \{0, 1, 2, \ldots\}$. We usually denote the symmetric generators $t_0, t_1, t_2, \ldots$ by their indices 0, 1, 2,...

In this thesis we use the technique of double coset enumeration to compute various homomorphics images of 2^{*7}: $L_3(2)$ factored by suitable relations.

Symmetric Generation of a Group

Now symmetric presentation for $2^{*7}: L_3(2)$ are given by $2^{*7}: L_3(2) \cong < x,y,t \mid x^7 = y^2 = (xy)^3 = [x,y]^4 = 1 = t^2 = [xy, t^{x^4}] = [y, t^{x^3}] >$ where $L_3(2) = <x,y>$, and $x \sim (0,1,2,3,4,5,6)$, $y \sim (2,6)(4,5)$. We show that

(1) $G = \dfrac{2^{*7}: L_3(2)}{[(0123456)t_0]^6, [(26)(45)t_0t_2]^4} \cong S_7$ (see chapter 2).

(2) $G = \dfrac{2^{*7}: L_3(2)}{[(0123456)t_0]^8} \cong S_8$ (see chapter 3).

(3) $G = \dfrac{2^{*7}: L_3(2)}{[(0123456)t_0]^6} \cong 3^{\bullet}S_7$ (see chapter 4).

(4) $G = \dfrac{2^{*7}: L_3(2)}{[(0123456)t_0]^7} \cong 2^{3_1}\, 2^{3_1+3_2}: L_3(2)$ (see chapter 5).

The main result of the project is to represent every element of J_3: 2, the automorphism group of the Janko sporadic group J_3, as a permutation of $L_2(16)$:4 on 120 letters followed by a word, in the t_i, of length at most 3.

In order to achieve this, we factor the progenitor 2^{*120}:

$L_2(16):4 \cong < x,y,z | \ x^{17} = y^8 = (x^y)*x^{-2} = z^2 = y^z*y^{-5} = (x*y*z)^4$

$= (x*z)^{17} = 1>$ by the relation $(x^3*y^5*z*t)^5 = 1$.

Where x ~ (2, 3, 5, 11, 12, 27, 25, 24, 10, 7, 18, 38, 41, 20, 21, 8, 4)(6, 14, 28, 54, 64, 100, 56, 55, 29, 23, 9, 22, 43, 78, 72, 36, 16)(13, 19, 40, 74, 37, 32, 62, 108, 84, 47, 45, 66, 34, 17, 15, 33, 30)(26, 50, 88, 97, 73, 99, 119, 114, 118, 120, 102, 70, 98, 113, 77, 42, 52)(31, 58, 96, 53, 68, 111, 110, 94, 117, 109, 65, 76, 115, 83, 46, 82, 60)(35, 67, 81, 44, 80, 63, 107, 87, 49, 86, 93, 92, 51, 91, 89, 61, 69)(39, 48, 85, 71, 112, 103, 57, 101, 116, 95, 90, 106, 59, 105, 104, 79, 75),

y ~ (3, 5, 12, 10, 4, 8, 20, 7)(6, 15, 28, 19, 9, 13, 29, 17)(11, 25, 41, 24, 21, 38, 27, 18)(14, 30, 56, 47, 23, 33, 64, 32)(16, 34, 36, 45, 22, 40, 43, 37)(26, 51, 76, 39, 42, 49, 53, 48)(31, 59, 99, 63, 46, 57, 102, 61)(35, 68, 71, 73, 44, 65, 79, 70)(50, 89, 82, 116, 77, 107, 60, 90)(52, 93, 94, 95)(54, 74, 72, 84, 55, 66, 78, 62)(58, 104, 120, 91, 83, 112, 119, 87)(67, 110, 101, 98, 81, 117, 106, 97)(69, 96, 75, 113, 80, 115, 85, 88)(86, 111, 103, 118, 92, 109, 105, 114)(100, 108),

z ~ (1, 2)(3, 6)(4, 9)(5, 13)(7, 19)(8, 15)(10, 17)(11, 26)(12, 28)(14, 31)(16, 35)(18, 39)(20, 29)(21, 42)(22, 44)(23, 46)(24, 48)(25, 49)(27, 53)(30, 57)(32, 63)(33, 59)(34, 65)(36, 71)(37, 73)(38, 51)(40, 68)(41, 76)(43, 79)(45, 70)(47, 61)(50, 77)(52, 94)(54, 97)(55, 98)(56, 99)(60, 82)(62, 101)(64, 102)(66, 67)(69, 109)(72, 110)(74, 81)(75, 114)(78, 117)(80, 111)(84, 106)(85, 118)(86, 88)(87, 91)(92, 113)(93, 95)(96, 103)(100, 108)(104, 112)(105, 115).

and obtain

$$G = \frac{2^{*120}: L_2(16):4}{[(x^3y^5z)t]^5} \cong J_3: 2 \text{ (see chapter 6).}$$

CHAPTER TWO

SYMMETRIC GENERATION OF S_7

Symmetric Presentation

Symmetric presentations for 2^{*7}: $L_3(2)$ are given by

$< x, y, t \mid x^7 = y^2 = (xy)^3 = [x,y]^4 = 1 = t^2 = [xy, t^{x^4}] = [y, t^{x^3}] >$

where $L_3(2) = <x,y>$, and $x \sim (0,1,2,3,4,5,6)$, $y \sim (2,6)(4,5)$. We factor by the relation.

$[(0\ 1\ 2\ 3\ 4\ 5\ 6)\ t_0]^6$ and $[(2\ 6)(4\ 5)\ t_0\ t_2]^4$

to obtain

$$G = \frac{2^{*7} : L_3(2)}{[(0123456)t_0]^6, [(26)(45)t_0t_2]^4} \cong S_7 \text{ (see [4])}.$$

Let $N = L_3(2)$. Now N is transitive on the set of symmetric generators $\omega = \{0,1,..,6\}$. The double coset Nt_iN contains 7 single cosets, since

$Nt_0N = \{Nt_0n \mid n \in N\} = \{Nnn^{-1}t_0n \mid n \in N\}$

$= \{Nnt_0^n \mid n \in N\} = \{Nt_0^n \mid n \in N\}$

$= \{Nt_{n(0)} \mid n \in N\} = \{Nt_0, Nt_1, Nt_2, Nt_3, Nt_4, Nt_5, Nt_6\}$

Now $N^0 = \langle (1, 2, 3, 4, 5, 6, 7), (2, 6)(4, 5) \rangle$

is transitive on $\{1,2,3,4,5,6\}$, and its orbits on ω are $\{0\}$, $\{1,2,3,4,5,6\}$.

The relation $[\pi t_0]^6 = 1$, where $\pi = (0\ 1\ 2\ 3\ 4\ 5\ 6)$ gives

$\pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0 = \pi^6\ \pi^{-5}\ t_0\ \pi^5\ \pi^{-4}\ t_0\ \pi^4\ \pi^{-3}\ t_0\ \pi^3$
$\pi^{-2}\ t_0\ \pi^2\ \pi^{-1}\ t_0\ \pi\ t_0$

$\Rightarrow\ [(0\ 1\ 2\ 3\ 4\ 5\ 6)\ t_0]^6 = [\pi\ t_0]^6 = \pi^6 t_5\ t_4\ t_3\ t_2\ t_1\ t_0$

Similarly, $[(1\ 2\ 4)(3\ 6\ 5)\ t_1]^6 = t_4\ t_2\ t_1\ t_4\ t_2\ t_1$,

$[(2\ 6)(4\ 5)\ t_2]^{12} = t_6\ t_2\ t_6\ t_2\ t_6\ t_2\ t_6\ t_2\ t_6\ t_2\ t_6\ t_2$,

$[(1\ 3)(2\ 4\ 6\ 5)\ t_1]^6 = (2\ 6)(4\ 5)\ t_3\ t_1\ t_3\ t_1\ t_3\ t_1$,

$[(2\ 6)(4\ 5)\ t_0\ t_2]^4 = t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2$

Now $t_4\ t_2\ t_1\ t_4\ t_2\ t_1 = e$, so we have $Nt_4\ t_2\ t_1\ t_4\ t_2\ t_1 = N$.

$Nt_4\ t_2\ t_1\ t_4\ t_2\ t_1(1340652) = N(1340652)$

$\Rightarrow\ N(1340652)\ t_4\ t_2\ t_1\ t_4\ t_2\ t_1^{(1340652)} = N$

$\Rightarrow\ N\ t_0\ t_1\ t_3\ t_0\ t_1\ t_3 = N,\ \Rightarrow\ t_3\ t_1 \sim t_0\ t_1\ t_3\ t_0$

$\Rightarrow\ t_0\ t_1 \sim t_3\ t_1\ t_0\ t_3$

$(2\ 6)(4\ 5) t_3\ t_1\ t_3\ t_1\ t_3\ t_1 = e$, so we have

$(2\ 6)(4\ 5) = t_3\ t_1\ t_3\ t_1\ t_3\ t_1 \Rightarrow N\ t_3\ t_1\ t_3\ t_1\ t_3\ t_1 = N$

$\Rightarrow\ t_1\ t_3 \sim t_3\ t_1\ t_3\ t_1$

and $N(2\ 6)(4\ 5)^{(1\ 0\ 3)(2\ 4\ 5)} = t_3\ t_1\ t_3\ t_1\ t_3\ t_1^{(1\ 0\ 3)(2\ 4\ 5)}$

(conjugate with elements of N)

$\Rightarrow\ (2\ 5)(4\ 6) = t_1\ t_0\ t_1\ t_0\ t_1\ t_0$

(since $(2\ 6)(4\ 5)^{(1\ 0\ 3)(2\ 4\ 5)} = (2\ 5)(4\ 6)$)

$\Rightarrow\ N\ t_1\ t_0\ t_1\ t_0\ t_1\ t_0 = N,\ \Rightarrow\ t_1\ t_0 \sim t_0\ t_1\ t_0\ t_1$,

$\Rightarrow\ t_1\ t_0\ t_1\ t_0 \sim t_0\ t_1$

and $(2\ 6)(4\ 5)^{(1\ 3\ 0)(2\ 5\ 4)} = t_3\ t_1\ t_3\ t_1\ t_3\ t_1^{(1\ 3\ 0)(2\ 5\ 4)}$

$\Rightarrow\ (2\ 4)(5\ 6) = t_0\ t_3\ t_0\ t_3\ t_0\ t_3$

[since $(2\ 6)(4\ 5)^{(1\ 3\ 0)(2\ 5\ 4)} = (2\ 4)(5\ 6)$]

$\Rightarrow N\ t_0\ t_3\ t_0\ t_3\ t_0\ t_3 = N$, $\Rightarrow t_0\ t_3\ t_0\ t_3 \sim t_3\ t_0$

$\Rightarrow t_0\ t_3 \sim t_3\ t_0\ t_3\ t_0$

We also use $t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2 = e$ to compute the following:

$N(2\ 5)(4\ 6)\ t_0\ t_1\ t_0\ t_1\ t_0\ t_1 = N(2\ 6)(4\ 5)t_1\ t_3\ t_1\ t_3\ t_1\ t_3$

$\Rightarrow 7\ 1\ 7\ 1\ \underline{7\ 1} \sim 1\ 3\ 1\ 3\ 1\ 3$, $\Rightarrow \underline{7\ 1\ 7\ 1} \sim 1\ 3\ 1\ 3\ 1\ 3\ 1\ 7$,

$\Rightarrow 1\ 7 \sim 1\ 3\ 1\ 3\ \underline{1\ 3\ 1\ 7}$, $\Rightarrow \underline{1\ 7} \sim 1\ 3\ 1\ 3\ 7\ 1\ 3\ 1$

(since $(2\ 5)(4\ 6)7\ 1\ 7 \sim 1\ 7\ \underline{1}$; $1\ 7\ 1\ 1 \sim 1\ 7$;

and $1\ 3\ 1\ 7\ \underline{1\ 3\ 1\ 7}$ is identity : $1317=7131$)

$\Rightarrow 1\ 3\ \underline{3\ 7} \sim 1\ 3\ 1\ 3\ 7\ 1\ 3\ 1$, $\Rightarrow 1\ 3 \sim \underline{1\ 3\ 1\ 3}\ 7\ 1\ 3\ 1\ 7\ 3$

[since $1\ \underline{3\ 1\ 3}\ 7\ 1\ 3\ 1\ 7\ 3 \sim 1\ (2\ 6)(4\ 5)1\ 3\ 1\ \ 7\ 1\ 3\ 1\ 7\ 3$

$\Rightarrow 1\ 3\ 1\ 3\ 7\ 1\ 3\ 1\ 7\ 3 \sim (2\ 6)(4\ 5)\underline{1^{(2\ 6)(4\ 5)}1}\ 3\ 1\ 7\ 1\ 3\ 1\ 7\ 3$,

$\Rightarrow 1\ 3\ 1\ 3\ 7\ 1\ 3\ 1\ 7\ 3 \sim 3\ 1\ 7\ 1\ 3\ 1\ 7\ 3$]

$\Rightarrow 1\ 3 \sim 3\ \underline{1\ 7\ 1\ 3}\ 1\ 7\ 3$

[Since $3\ \underline{1\ 7\ 1}\ 3\ 1\ 7\ 3 \sim 3\ \underline{(2\ 5)(4\ 6)7\ 1\ 7}\ 3\ 1\ 7\ 3$

$\Rightarrow 3\ \underline{1\ 7\ 1}\ 3\ 1\ 7\ 3 \sim 3\ 7\ 1\ 7\ 3\ 1\ 7\ 3$]

$\Rightarrow 1\ 3 \sim 3\ 7\ \underline{1\ 7\ 3\ 1}\ 7\ 3 \Rightarrow\ \ 1\ 3 \sim 3\ 7\ \underline{3\ 7\ 7\ 3}$

$\Rightarrow 13 \sim 3\ 7$

Similarly,

$N(2\ 4)(5\ 6)\ t_3\ t_7\ t_3\ t_7\ t_3\ t_7 = N(2\ 6)(4\ 5)t_1\ t_3\ t_1\ t_3\ t_1\ t_3$

$\Rightarrow 3\ 7\ 3\ 7\ \underline{3\ 7} \sim 1\ 3\ 1\ 3\ 1\ 3 \Rightarrow \underline{3\ 7\ 3\ 7} \sim 1\ 3\ 1\ 3\ 1\ 3\ 7\ 3$

$\Rightarrow 7\ 3 \sim 1\ 3\ 1\ \underline{3\ 1\ 3\ 7}\ 3;\ 7\ \underline{3} \sim 1\ 3\ 1\ 7\ 3\ 1\ 3\ 3$

[since 3737 ~ 73, and 3137 = 7313]

$\Rightarrow \underline{7} \sim 1\ 3\ 1\ 7\ 3\ 1\ 3 \Rightarrow 7\ 1 \sim \underline{1\ 3\ 1}\ 7\ 3\ \underline{1\ 3\ 1}$

(since 13 = 3131 and 131 ~ 313)

$7\ 1 \sim 3\ 1\ 3\ 7\ \underline{3\ 3}\ 1\ 3,\ 7\ 1 \sim 3\ \underline{1\ 3\ 7\ 1}\ 3,$

$\Rightarrow 7\ 1 \sim 3\ 7\ \underline{3\ 3}$ (since 1371 = 73).

Thus,

$7\ 1 \sim 3\ 7 \sim 1\ 3.$

$[(0\ 1\ 2\ 3\ 4\ 5\ 6)\ t_0]^6 = (0\ 6\ 5\ 4\ 3\ 2\ 1)\ t_5\ t_4\ t_3\ t_2\ t_1\ t_0 = e$

$\Rightarrow N(0\ 6\ 5\ 4\ 3\ 2\ 1)\ t_5\ t_4\ t_3 = N\ t_0\ t_1\ t_2$

$\Rightarrow t_5\ t_4\ t_3 \sim t_0\ t_1\ t_2$ or $5\ 4\ 3 \sim 7\ 1\ 2$

$[(0\ 6\ 5\ 4\ 3\ 2\ 1)\ t_5\ t_4\ t_3]^{(1\ 3\ 4\ 0\ 6\ 5\ 2)} = [t_0\ t_1\ t_2]^{(1\ 3\ 4\ 0\ 6\ 5\ 2)}$

$N\ t_5\ t_4\ t_3{}^{(1\ 3\ 4\ 0\ 6\ 5\ 2)} = N\ [t_0\ t_1\ t_2]^{(1\ 3\ 4\ 0\ 6\ 5\ 2)},$

$N\ (1\ 3\ 4\ 0\ 6\ 5\ 2)\ t_2\ t_0\ t_4 = N\ t_6\ t_3\ t_1$

$\Rightarrow 2\ 7\ 4 \sim 6\ 3\ 1$ since $46 \sim 34 \sim 63 \Rightarrow 2\ 7\ 4 \sim 4\ 6\ 1$

Similarly,

$[(0\ 6\ 5\ 4\ 3\ 2\ 1)\ t_5\ t_4\ t_3]^{(1\ 4\ 6\ 2\ 3\ 0\ 5)} = [t_0\ t_1\ t_2]^{(1\ 4\ 6\ 2\ 3\ 0\ 5)}$

$\Rightarrow 1\ 6\ 7 \sim 5\ 4\ 3$ since $t_1\ t_6\ t_0 \sim t_5\ t_4\ t_3$

$[(0\ 6\ 5\ 4\ 3\ 2\ 1)\ t_5\ t_4\ t_3]^{(1\ 0\ 2\ 4\ 5\ 3\ 6)} = [t_0\ t_1\ t_2]^{(1\ 0\ 2\ 4\ 5\ 3\ 6)}$

$\Rightarrow 3\ 5\ 6 \sim 5\ 4\ 3$ since $t_3\ t_5\ t_6 \sim t_5\ t_4\ t_3$

$[(0\ 6\ 5\ 4\ 3\ 2\ 1)\ t_5\ t_4\ t_3]^{(1\ 6\ 3\ 5\ 4\ 2\ 0)} = [t_0\ t_1\ t_2]^{(1\ 6\ 3\ 5\ 4\ 2\ 0)}$

$\Rightarrow 4\ 2\ 5 \sim 1\ 6\ 7$ since $t_4\ t_2\ t_5 \sim t_1\ t_6\ t_0$

$\Rightarrow$ 1 4 5 ~ 1 6 7 since 14 ~ 42 ~ 21

Thus,

5 4 3 ~ 7 1 2 ~ 2 7 4 ~ 4 6 1 ~ 1 6 7 ~ 3 5 6 ~ 1 4 5.

Summarize:

The set of all double cosets $[\omega] = N\omega N$, coset stabilizing subgroups $N^{(\omega)}$, and the number of single cosets in each double coset are displayed below.

Table 2.1. The Double Coset $N\omega N = [\omega]$,

where $N = L_3(2)$, $\omega = \{0,1,2,3,4,5,6\}$

$[\omega]$	Coset Stabilising subgroup $N^{(\omega)}$	Number of cosets in $[\omega]$
[*]	N is transitive on $\omega = \{0,1,2,3,4,5,6\}$	1
[0]	$N^{(0)} = N^0 \cong \langle(1,2,3,6)(4,5), (2,6)(4,5), (1,3)(4,5), (1,5)(3,4), (2,4)(5,6)\rangle$, $\lvert N^{(0)}\rvert = 24$, $N^{(0)}$ has orbits {0} and {1,2,3,4,5,6} on ω	7
[01]	$N^{01} \cong \langle(2,6)(4,5), (2,4)(5,6)\rangle$. Now 1 3 ~3 7 ~ 7 1 $\Rightarrow N^{(01)} \cong \langle N^{01}$, (1,3,7)(4,5,6), (1,3,7)(2,6,5), (1,3,7)(2,4,6), (1,3,7)(2,5,4), (1,7,3)(2,4,5), (1,7,3)(2,6,4), (1,7,3)(4,6,5), (1,7,3)(2,5,6)$\rangle$ Now $\lvert N^{(01)}\rvert = 12$ and $N^{(01)}$ has orbits {0,1,3} and {2,4,5,6}	14
[012]	$N^{012} = \langle e\rangle$. But 5 4 3 ~ 7 1 2 ~ 2 7 4 ~ 4 6 1 ~ 1 6 7 ~ 3 5 6 ~ 1 4 5. Thus $N^{(012)} \cong \langle$(1,6,3,5,4,2,7), (1,5,7,3,2,6,4), (1,4,7)(2,5,3),	8

	(1,7,2,4,5,3,6), (1,4,6,2,3,7,5), (1,6,2)(4,5,7)> Now $\|N^{(012)}\|$ = 21 and $N^{(012)}$ is transitive on ω	

It is readily verified that

S_7 = <(1 4)(2 3 5)(6 7),(4 5 6),(1 2)(3 6)(4 7)> ≅

$<t_0t_1t_2t_3t_4t_5t_6,\ t_0> = G$

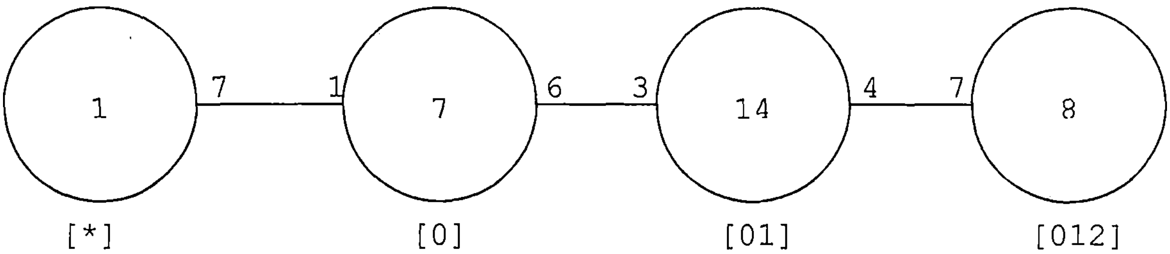

Figure 2.1. Cayley Graph of S_7 over $L_3(2)$

From the above graph, we compute t_0, t_1, t_2, t_3, t_4, t_5, and t_6:

t_0: (* 0)(1 31)(6 60)(2 20)(5 45)(3 01)(21 254)(56 251)(4 05)(16 053)(32 051)(12 453)(34 654)(43 153)(23 653)

t_1: (* 1)(0 01)(6 56)(2 21)(31 3)(60 653)(5 16)(20 645)(4 12)(45 153)(32 251)(254 43)(05 051)(34 053)(23 453)

t_2: (* 2)(0 60)(1 12)(6 20)(31 153)(5 23)(3 32)(21 4)(01 053)(56 051)(45 645)(16 254)(251 05)(34 453)(43 653)

t_3: (* 3)(0 31)(1 01)(6 34)(2 23)(60 254)(5 32)(20 251)(21 051)(56 153)(4 43)(45 453)(16 653)(12 654)(05 053)

t_4: (* 4)(0 45)(1 21)(6 43)(2 12)(31 653)(60 051)(5 05)(3 34)(20 053)(01 251)(56 453)(16 654)(32 254)(23 153)

t_5: (* 5)(0 05)(1 56)(6 16)(2 32)(31 654)(60 453)(3 23)(20 153)(21 053)(01 254)(4 45)(12 653)(251 43)(34 051)

t_6: (* 6)(0 20)(1 16)(2 60)(31 051)(5 56)(3 43)(21 153)(01 453)(4 34)(45 254)(32 654)(12 251)(05 653)(053 23)

The Identification of Cosets Labeling

MAGMA Labeling	Single coset		
1	[*]	11	[2, 1]
2	[7]	12	[7, 1]
3	[1]	13	[5, 6]
4	[6]	14	[4]
5	[2]	15	[4, 5]
6	[3, 1]	16	[1, 6]
7	[6, 7]	17	[3, 2]
8	[5]	18	[2, 5, 4]
9	[3]	19	[1, 2]
10	[2, 7]	20	[2, 5, 1]

21	[7, 5]	26	[2, 3]
22	[3, 4]	27	[4, 5, 3]
23	[7, 5, 3]	28	[6, 5, 3]
24	[4, 3]	29	[6, 5, 4]
25	[7, 5, 1]	30	[1, 5, 3

Define (t_i):

(t_0):

(1, 2)(3, 6)(4, 7)(5, 10)(8, 15)(9, 12)(11, 18)(13, 20)(14, 21)(16, 23)(17, 25)(19, 27)(22, 29)(24, 30)(26, 28)

(t_1):

(1, 3)(2, 12)(4, 13)(5, 11)(6, 9)(7, 28)(8, 16)(10, 29)(14, 19)(15, 30)(17, 20)(18, 24)(21, 25)(22, 23)(26, 27)

(t_2):

(1, 5)(2, 7)(3, 19)(4, 10)(6, 30)(8, 26)(9, 17)(11, 14)(12, 23)(13, 25)(15, 29)(16, 18)(20, 21)(22, 27)(24, 28)

(t_3):

(1, 9)(2, 6)(3, 12)(4, 22)(5, 26)(7, 18)(8, 17)(10, 20)(11, 25)(13, 30)(14, 24)(15, 27)(16, 28)(19, 29)(21, 23)

(t_4):

(1, 14)(2, 15)(3, 11)(4, 24)(5, 19)(6, 28)(7, 25)(8, 21)(9, 22)(10, 23)(12, 20)(13, 27)(16, 29)(17, 18)(26, 30)

(t_5):

(1, 8)(2, 21)(3, 13)(4, 16)(5, 17)(6, 29)(7, 27)(9, 26)(10, 30)(11, 23)(12, 18)(14, 15)(19, 28)(20, 24)(22, 25)

(t_6):

(1, 4)(2, 10)(3, 16)(5, 7)(6, 25)(8, 13)(9, 24)(11, 30)(12, 27)(14, 22)(15, 18)(17, 29)(19, 20)(21, 28)(23, 26)

The group is defined by the symmetric presentation. It index is at most:

$$\frac{|N|}{|N|}+\frac{|N|}{|N^{(0)}|}+\frac{|N|}{|N^{(01)}|}+\frac{|N|}{|N^{(012)}|} = 1 + 7 + 14 + 8 = 30$$

(According to the S_7 graph)

The index of N in G has most 30. $\Rightarrow$ the order of G is at most $30|N| = 30 \times 168 = 5040$. (At this point G has order at most $|L_3(2)|(1+7+14+8) = 168\times30 = 5040$. Since $|L_3(2)|$ is 168)

Thus, the order of G is most at 5040.

Since the group S_7 is generated by x, y, and t

$\Rightarrow$ S_7 is an image of G.

$\Rightarrow |G| \geq |S_7| \Rightarrow |G| \geq 5040$

$\Rightarrow 5040 \leq |G| \leq 5040 \Rightarrow |G| = 5040$

$\Rightarrow G \cong S_7$

CHAPTER THREE

SYMMETRIC GENERATION OF S_8

Symmetric Presentation

Symmetric presentations 2^{*7}: $L_3(2)$ are given by

$< x,y,t \mid x^7 = y^2 = (xy)^3 = [x,y]^4 = 1 = t^2 = [xy, t^{x^4}] = [y, t^{x^3}]$ $>$, where $L_3(2) = <x,y>$, and the action of N on the symmetric generators is given by $x \sim (0,1,2,3,4,5,6)$, $y \sim (2,6)(4,5)$. We factor G by using this relation $[(0\ 1\ 2\ 3\ 4\ 5\ 6)\ t_0]^8$ to obtain

$$G = \frac{2^{*7} : L_3(2)}{[(0123456)t_0]^8} \cong S_8 \text{ (see [4])}.$$

Let $N = L_3(2)$. Now N is transitive on the symmetric generators $\omega = \{0,1,..,6\}$. The double coset Nt_iN contains 7 single cosets. Now $N^0 = \langle (1,2,3,4,5,6,7), (2,6)(4,5) \rangle$ is transitive on $\{1,2,3,4,5,6\}$, and its orbits on ω are $\{0\}$, $\{1,2,3,4,5,6\}$.

The relation $[\pi t_0]^8 = 1$, where $\pi = (0\ 1\ 2\ 3\ 4\ 5\ 6)$ gives

$\pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0 = \pi^8\ \pi^{-7}\ t_0\ \pi^7\ \pi^{-6}\ t_0\ \pi^6\ \pi^{-5}\ t_0\ \pi^5\ \pi^{-4}\ t_0\ \pi^4\ \pi^{-3}\ t_0\ \pi^3\ \pi^{-2}\ t_0\ \pi^2\ \pi^{-1}\ t_0\ \pi\ t_0$

$\Rightarrow [(0\ 1\ 2\ 3\ 4\ 5\ 6)\ t_0]^8 = [\pi\ t_0]^8 = \pi\ t_0\ t_6\ t_5\ t_4\ t_3\ t_2\ t_1\ t_0$

We also use other relations,

$[(1\ 2\ 4)(3\ 5\ 6)\ t_1]^{12} = t_4\ t_2\ t_1\ t_4\ t_2\ t_1\ t_4\ t_2\ t_1\ t_4\ t_2\ t_1$

$[(1\ 2\ 4)(3\ 5\ 6)\ t_3]^{12} = t_5\ t_6\ t_3\ t_5\ t_6\ t_3\ t_5\ t_6\ t_3\ t_5\ t_6\ t_3$

$[(2\ 6)(4\ 5)\ t_2]^6 = t_6\ t_2\ t_6\ t_2\ t_6\ t_2$

$[(1\ 3)(2\ 4\ 6\ 5)\ t_1]^{12} = t_3\ t_1\ t_3 t_1\ t_3\ t_1\ t_3\ t_1\ t_3\ t_1\ t_3\ t_1$

$[(1\ 3)(2\ 4\ 6\ 5)\ t_2]^{10} = (2\ 6)(4\ 5)\ t_5\ t_6\ t_4\ t_2\ t_5\ t_6\ t_4\ t_2\ t_5\ t_6$
$t_4\ t_2$

$[(2\ 6)(4\ 5)\ t_0\ t_2]^4 = t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2$

$[(2\ 6)(4\ 5)\ t_1\ t_2]^4 = t_1\ t_6\ t_1\ t_2\ t_1\ t_6\ t_1\ t_2$

Since 525252 is identity (e),

by using relation, $[t_6\ t_2\ t_6\ t_2\ t_6\ t_2]$ conjugates to element,

n, in N.

$e = 5\underline{25}252 = 527\underline{7525}2 = 52\underline{75}2572 = 5271\underline{152}572 = 5\underline{27}1525172$

$= 521\underline{171}525172 = 5215\underline{171}25172 = 5215\underline{7}1725172$

$= 52\underline{17}5751725172 = 5212\underline{2757}51725172$

$= \underline{5212757}251725172 = (12)(57)\underline{(12)(57)\ 5212757}251725172$

$= (12)(57)712157\ (\underline{12)(57)\ 7251725172} = (12)(57)712157\ \underline{e}$

$\Rightarrow e = (12)(57)712\underline{157} \quad \Rightarrow 751 = (12)(57)712$

For $e = (12)(57)712157$

$\Rightarrow e^{(512)(346)} = \underline{(12)(57)712157^{\ (512)(346)}}$

$\Rightarrow e = (25)(17)\ 725\underline{217} \quad \Rightarrow 712 = (25)(17)\ 725$

Thus,

$751 \sim 712 \sim 725$

Since $e = 717171 \Rightarrow 717 = 171$

$e = 6464\underline{64}$ and $e = 141414 \Rightarrow e = 6464\underline{6141}41\underline{44}$

$= 646414\underline{16}41 = 646414176767\underline{66}41 = 64641\underline{4767}16741$

$= 646\underline{4171}167416741 = 64617\underline{14}167416741$

$= \underline{646171\ (17)(46)}(17)(46)\ 4167416741$

$= (17)(46)\ 464717\ \underline{(17)(46)\ 4167416741}$

Since $(26)(45)\ 4256425642 = e$

$\Rightarrow\ (26)(45)\ 4256425642^{(12)(3567)} = (17)(46)\ 4167416741 = e$

$\Rightarrow\ e = (17)(46)\ 464\underline{717} \Rightarrow 717 = (17)(46)\ 464$

For $e = \underline{21}2121 \Rightarrow e = 25\underline{5121}21 = 25121\underline{5}21 = 2512\underline{1757}5721$

Since $757575 = e \Rightarrow 75757 = 5$

$\Rightarrow e = 251\underline{2757}15721 = 2\underline{51}757215721 = 2521\underline{1217}57215721$

$= 25217\underline{12}157215721 = \underline{252171\ (17)(25)}(17)(25)\ 2157215721$

$= (17)(25)\ 525717\ \underline{(17)(25)\ 2157215721}$

Since $(26)(45)\ 4256425642 = e$

$\Rightarrow (26)(45)\ 4256425642^{(142)(367)} = (17)(25)\ 2157215721 = e$

$\Rightarrow e = (17)(25)\ 525\underline{717} \Rightarrow 717 = (17)(25)\ 525$

Since $717 = (17)(46)\ 464$ and $646464 = e$

$\Rightarrow 646 = 464$

$\Rightarrow 717 = (17)(46)\ 646$

Since $717 = (17)(25)\ 525$ and $525252 = e$

$\Rightarrow 525 = 252$

$\Rightarrow 717 = (17)(25)\ 252$

Thus,

$717 \sim 171 \sim 464 \sim 525 \sim 646 \sim 252$

Since $131 = (13)(26)\ 262$

$\Rightarrow 131^{(137)(246)} = (13)(26)\ 262^{(137)(246)}$

$\Rightarrow 373 = (37)(24)\ 424$

$\Rightarrow e = (24)(37)\ 37\underline{3424} = (24)(37)\ 3\underline{74}243$

$= (24)(37)\ 376\underline{6424}3 = (24)(37)\ \underline{37}642463$

$= (24)(37)\ 36\underline{6764}2463 = \underline{(24)(37)}\ 3646762463$

$= (13)(26)\underline{(173)(264)}\ 3646762463$

$= (13)(26)\ 7262\underline{124627\ (173)(264)} = e$

$\Rightarrow (137)(246)\ 726421 = (13)(26)\ 7262 = \underline{7131}$

(by solving relation)

$\Rightarrow 7132 = (137)(246)\ 7264$

Since $e = 5474\underline{4745}$, $\Rightarrow e = 5474\underline{5}474 = 54745232\underline{2324}74$

$= 547\underline{4232}5423274 = 547232\underline{4547}232554 = 5\underline{4232}7454752327754$

$= 523247454757232754$

[Since $7131 = (13)(45)\ 7454$

$\Rightarrow 7131^{(12)(57)} = (13)(45)\ 7454^{(12)(57)}$

$\Rightarrow 5232 = \underline{(23)(47)}\ 5474 \Rightarrow (23)(47) = 52324745$]

$\Rightarrow e = \underline{(23)(47)}\ 4757232754$

$\Rightarrow e = (13)(45)\underline{(123)(457)}\ 4757\underline{232754}$

$\Rightarrow (13)(24)7545\ \underline{(123)(457)} = 457232$

$\Rightarrow (13)(24)7545 = (132)(475)\ 745121$

Since $7131 = (13)(45)\ 7545 \Rightarrow 7131 = (132)(475)\ 745121$

$\Rightarrow 7132 = (132)(475)\ 7451$

We have $725 = 725$ and $(7361425)\ 725 = \underline{(7361425)}\ 725$

$\Rightarrow (7361425)\ 725 = (3657)(14)\underline{(15)(27)\ 725} = (3657)(14)\ 751$

Since $(1257)(34)\ 753^{(13)(26)} = (3657)(14)\ 751$

and $(7361425)\ 725 = (7123456)\ 765^{(13)(26)}$

So $(1257)(34)\ 753^{\underline{(13)(26)}} = (7123456)\ 765^{\underline{(13)(26)}}$

$\Rightarrow (1257)(34)\ 753 = \underline{(7123456)\ 765}$

$\Rightarrow (1257)(34)\ 753\underline{4} = (7123456)\ 765\underline{4}$

Since $7123 = (7123456)\ 7654$ (by relation),

$\Rightarrow (1257)(34)\ 7534 = 7123;\ (1257)(34)\ \underline{7534} = \underline{7123}$

$\Rightarrow (1257)(34)\ 7534^{(165)(243)} = 7123^{(165)(243)}$

$\Rightarrow \underline{(1764)(23)}\ 7123 = 7642 \Rightarrow 7123 = (1764)(23)\ 7642$

Thus,

$7123 \sim 7534 \sim 7642;\ 7132 \sim 7264 \sim 7451$

$e = 121212 = 16\underline{621}212 = 1621\underline{26}12 = 16\underline{2123}3612 = 16\underline{32}123612$

$= 16\underline{23}23123612 = \underline{16266323}123612 = 6261\underline{32}36123612$

$= 626\underline{131}1236123612 = 6263131236123612$

$= (13)(26)\underline{(13)(26)\ 626313}1236123612$

$= (13)(26)\ 262131\ \underline{(13)(26)\ 1236123612}$

Since $(26)(45)\ 4256425642 = e = (26)(45)\ 4256425642^{(14)(35)}$

$= (13)(26)\ 1236123612 \Rightarrow e = (13)(26)\ 262131$

$7 = 7 \Rightarrow 7 = 7\ (13)(26)\ 262\underline{131}$

$\Rightarrow$ 7 = (13)(26) 7262$\underline{131}$ $\Rightarrow$ 7131 = (13)(26) 7262

Since 313 = 131 $\Rightarrow$ 7131 = 7313

Since 626 = 262, $\Rightarrow$ 7131 = (13)(26) 7626

Since $7131^{(24)(56)}$ = (13)(26) $7262^{(24)(56)}$

$\Rightarrow$ 7131 = (13)(45) 7454

Since 454 = 545, $\Rightarrow$ 7131 = (13)(45) 7545

Thus,

7131 ~ 7313 ~ 7262 ~ 7626 ~ 7454 ~ 7545

Since e = 3737$\underline{37}$ $\Rightarrow$ e = 37$\underline{373}$117 = 37$\underline{13}$7313 = 3712$\underline{2373}$17

= 3$\underline{7123}$73217 = $\underline{3\ (1234567)}$ 7654$\underline{73217}$

[Since e = (1234567) 76543217 $\Rightarrow$ (1234567) 7654 = 7123]

$\Rightarrow$ 71237 = (1234567) 47654

Since e = 3737$\underline{37}$ $\Rightarrow$ e = 37$\underline{373}$667 = 37637$\underline{36}$7 = 376$\underline{3731}$167

= 3761$\underline{3736}$6167 = 37613$\underline{73}$67616 = 37613723$\underline{3236}$7616

= 376137236$\underline{3237}$616 = 376137236$\underline{73}$23616

= 37613723675$\underline{5232}$161 = 37613723675232$\underline{5161}$

= $\underline{376137236752321615}$ = (16)(23)$\underline{(16)(23)\ 3761372367}$52321615

= (16)(23) 2716273217 $\underline{(16)(23)\ 52321615}$

[Since 7131 = (13)(26) 7262

$\Rightarrow$ $7131^{(36)(57)}$ = (13)(26) $7262^{(36)(57)}$

$\Rightarrow$ 5161 = (16)(23) 5232 $\Rightarrow$ e = (16)(23) 52321615]

$\Rightarrow$ e = (16)(23) 2716273217 e

⇒ 71237 = (16)(23) 27162

Since e = 313131171717

⇒ e = 313132271717 = 3515153132717217

= 3515152313717217 = 3515152317173217 = 3512515317173217

= 3512351517173217 = 35123157173217 = 35123153717217

= 3512315321 = 35123153217117 = 35123153171217

= 35123151713217 = 351235661511713217

= 3512356151336717 3217 = 3512356315136717 3217 = e

⇒ 71237 = 351235631513671 = (12)(36) 13671

[Since 71237 = (16)(23) 27162

⇒ $71237^{(236)(475)}$ = (16)(23) $27162^{(236)(475)}$

51365 = (12)(36) 35123 ⇒ (12)(36) = 3512356351]

⇒ 71237 = (12)(36) 13671

Since e = 72121217 ⇒ e = 7244121217 = 227241214217

= 242721214217 = 241272214217 = 24127377314217

= 24173727314217 = 24737127314217 = 27374127314217

= 2763363741273 14217 = 276373634123637314217

= 27637412363731 4217 = 276374123613734217

= 27637412361437 3217 = (124)(376) 3614373217

[Since 7132 = (132)(475) 7451

⇒ $7132^{(1723654)}$ = (132)(475) $7451^{(1723654)}$

2763 = (124)(376)2147 ⇒ (124)(376) = 27637412]

$\Rightarrow 71237 = (124)(376)\ 36143$

Since $e = 767\underline{7673}53535 \Rightarrow e = 7673\underline{7675}3535 = 76735\underline{7673}535$

$= 767\underline{3537}67535 = 37675\underline{3}767535 = 3767\underline{5131}31767535$

$= 376\underline{7131}531767535 = 3761317531\underline{765357}$

$\Rightarrow 753567 = 3761317531$

[Since $751 = (15)(27)\ 725$

$\Rightarrow 751^{(13)(26)} = (15)(27)\ 725^{(13)(26)}$

$\Rightarrow 753 = (35)(67)765 \Rightarrow (35)(67) = 753567$]

$\Rightarrow \underline{(35)(67)} = 3761317531$

$\Rightarrow (167)(235)\underline{(16)(23)} = 3761317531$

$(167)(235) = (16)(23)\ 27162\underline{67526}$

$\Rightarrow (167)(235)\ 62576 = \underline{(16)(23)\ 27162}$

Since $71237 = (16)(23)\ 27162$

$\Rightarrow (167)(235)\ 62576 = (16)(23)\ 27162$

$\Rightarrow e = (167)(235)\ 62576\underline{73217} \Rightarrow 71237 = (167)(235)\ 62576$

Since $e = 3\underline{73}737 \Rightarrow e = 372\underline{2373}7 = 37237\underline{3121}217$

$= 3723\underline{7121}3217 = 372312\underline{1715}513217 = 3723125\underline{1715}13217$

$= 37231251\underline{7515}3217 = 37231\underline{2515}1573217 = 3723\underline{15}1521573217$

$= 372165\underline{5651}3521573217 = \underline{3}72165\underline{1565}3521573217$

$= 7\underline{7372}1651356552157321 7= 721\underline{7376}5135621573217$

$= \underline{7216737513}5621573217 = (165)(237)\ 5621573217 = e$

[Since $71237 = (167)(235)\ 62576$

$\Rightarrow 71237^{(12)(36)} = (167)(235)\ 62576^{(12)(36)}$

$\Rightarrow$ 72167 = (165)(237)31573 $\Rightarrow$ (165)(237) = 7216737513]

$\Rightarrow$ 71237 = (165)(237) 56215

Thus,

71237 = (1234567) 47654 = (16)(23) 27162

= (124)(376) 36143 = (167)(235) 62576 = (165)(237) 56215

= (12)(36) 13671

$\Rightarrow$ 71237 ~ 47654 ~ 27162 ~ 36143 ~ 62576 ~ 56215 ~ 13671

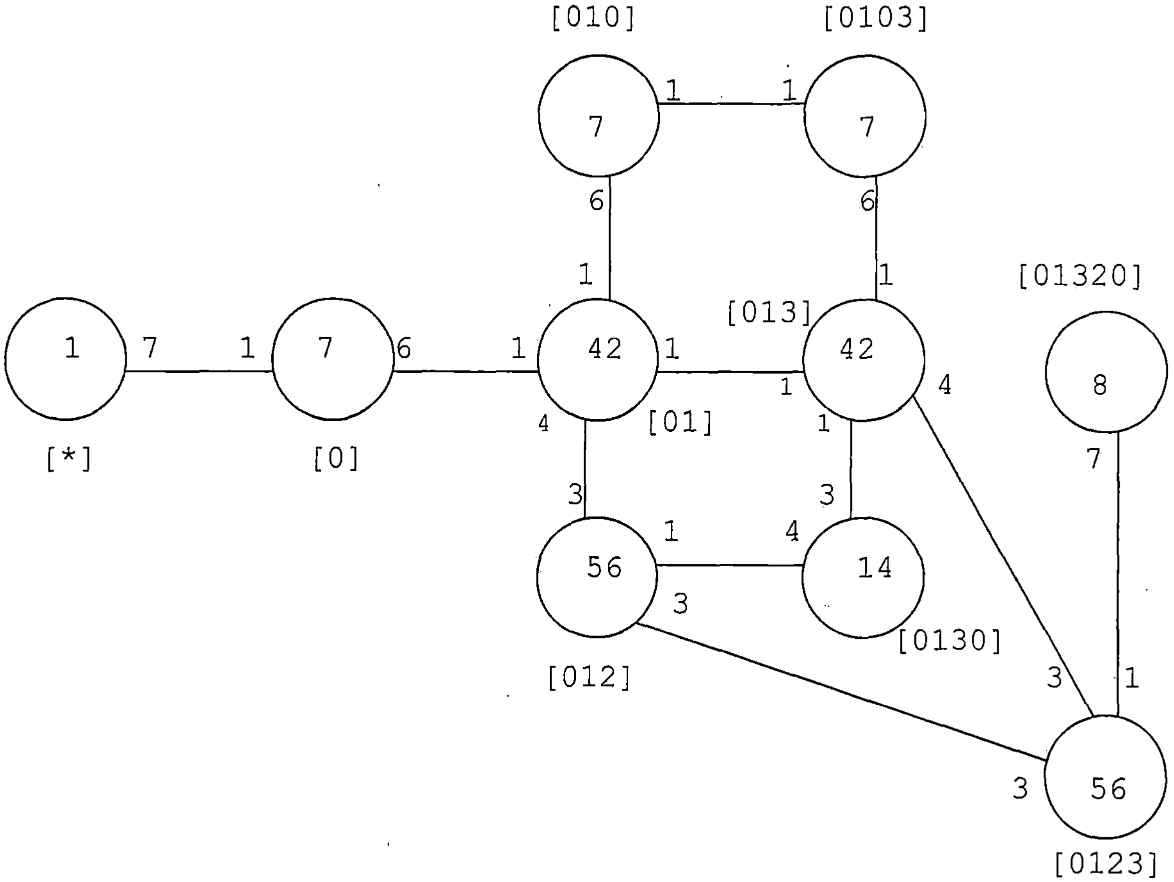

Figure 3.1. Cayley Graph of S_8 over $L_3(2)$

The Identification of Cosets Labeling

MAGMA Labeling	Single coset		
1	[*]	21	[6, 1]
2	[7]	22	[2, 5, 1]
3	[1]	23	[7, 2]
4	[6]	24	[6, 7, 6]
5	[2]	25	[6, 5]
6	[1, 7]	26	[1, 2]
7	[6, 7]	27	[7, 1, 7]
8	[5]	28	[4, 2]
9	[3]	29	[5, 6, 7]
10	[2, 7]	30	[4, 5]
11	[2, 1]	31	[4, 7]
12	[7, 6]	32	[4, 6]
13	[7, 1]	33	[4, 1]
14	[5, 6]	34	[2, 6]
15	[4]	35	[3, 1, 7]
16	[5, 7]	36	[1, 6, 7]
17	[3, 7]	37	[7, 5]
18	[3, 1]	38	[4, 3]
19	[1, 6]	39	[3, 6]
20	[3, 2]	40	[3, 4, 2]

41	[6, 4, 1]	64	[6, 2]
42	[3, 1, 6]	65	[2, 6, 7]
43	[1, 4, 7]	66	[1, 5]
44	[1, 3]	67	[4, 2, 1]
45	[2, 7, 2]	68	[2, 7, 6]
46	[2, 4, 2]	69	[2, 7, 1]
47	[2, 4]	70	[7, 5, 6]
48	[6, 3, 5]	71	[7, 4]
49	[5, 4]	72	[6, 4]
50	[2, 3]	73	[4, 2, 3]
51	[1, 2, 7]	74	[3, 5, 6]
52	[1, 2, 1]	75	[2, 5]
53	[5, 3]	76	[4, 5, 3]
54	[4, 2, 7]	77	[2, 3, 1]
55	[6, 7, 1]	78	[7, 5, 2]
56	[4, 5, 6]	79	[5, 3, 7]
57	[4, 5, 7]	80	[3, 1, 2]
58	[3, 4]	81	[3, 4, 5, 6]
59	[5, 1]	82	[1, 5, 7]
60	[5, 2]	83	[1, 4]
61	[4, 6, 7]	84	[7, 5, 3]
62	[3, 5]	85	[1, 3, 7]
63	[4, 6, 1]	86	[6, 2, 6]

87	[2, 3, 2]	110	[5, 3, 2]
88	[7, 3]	111	[4, 2, 5, 1]
89	[2, 3, 4]	112	[6, 7, 2]
90	[7, 4, 6]	113	[6, 7, 6, 2]
91	[5, 2, 4]	114	[1, 6, 5]
92	[5, 4, 7]	115	[7, 4, 2]
93	[6, 3]	116	[2, 1, 4, 2]
94	[2, 7, 4]	117	[6, 4, 5]
95	[7, 1, 6]	118	[5, 3, 4]
96	[5, 3, 1]	119	[2, 4, 5]
97	[7, 1, 2]	120	[4, 5, 3, 7]
98	[5, 6, 4]	121	[6, 3, 1]
99	[4, 2, 7, 1]	122	[2, 5, 4, 3]
100	[3, 4, 5]	123	[1, 6, 3]
101	[5, 6, 1]	124	[3, 4, 6, 3]
102	[3, 4, 6]	125	[4, 6, 5]
103	[3, 7, 2]	126	[3, 1, 2, 7]
104	[5, 1, 7]	127	[2, 3, 4, 5]
105	[5, 2, 7]	128	[2, 6, 1]
106	[3, 6, 5]	129	[7, 4, 3]
107	[6, 2, 7]	130	[1, 2, 4, 1]
108	[3, 7, 1]	131	[6, 4, 2]
109	[1, 5, 6]	132	[2, 4, 1]

133	[7, 2, 6]
134	[6, 2, 6, 7]
135	[1, 2, 3]
136	[2, 3, 5, 2]
137	[5, 2, 7, 3]
138	[4, 1, 3]
139	[6, 5, 1]
140	[4, 3, 6]
141	[6, 7, 5]
142	[3, 5, 4]
143	[3, 2, 5, 3]
144	[5, 4, 3, 7]
145	[4, 3, 6, 4]
146	[5, 6, 7, 1]
147	[3, 1, 4, 7]
148	[5, 6, 4, 1]
149	[3, 5, 2]
150	[3, 5, 6, 2]
151	[2, 3, 5]
152	[6, 3, 2]
153	[2, 5, 4]
154	[7, 3, 1]
155	[5, 1, 6]
156	[4, 1, 2]
157	[7, 1, 7, 3]
158	[1, 4, 2]
159	[1, 5, 3, 6]
160	[7, 4, 5]
161	[6, 4, 3]
162	[5, 3, 4, 2]
163	[5, 3, 6, 2]
164	[7, 1, 3]
165	[2, 7, 2, 6]
166	[2, 4, 2, 1]
167	[1, 2, 4]
168	[1, 6, 3, 5]
169	[7, 5, 4]
170	[1, 5, 3]
171	[5, 6, 1, 5]
172	[3, 1, 7, 3]
173	[6, 3, 1, 4]
174	[5, 3, 7, 2]
175	[2, 6, 4, 7]
176	[3, 4, 2, 6]
177	[6, 4, 5, 3]
178	[3, 6, 5, 4]

179	[1, 4, 3, 2]	202	[3, 1, 5, 7]
180	[1, 6, 7, 5]	203	[6, 5, 4, 1]
181	[4, 5, 7, 4]	204	[4, 3, 2, 6]
182	[4, 1, 3, 2]	205	[7, 5, 4, 7]
183	[4, 2, 3, 1]	206	[6, 7, 1, 2]
184	[2, 7, 1, 6]	207	[6, 7, 5, 2]
185	[6, 3, 5, 4]	208	[4, 6, 3]
186	[2, 1, 4]	209	[6, 3, 4]
187	[1, 2, 3, 4]	210	[2, 7, 4, 6]
188	[2, 6, 1, 7]	211	[2, 3, 1, 5]
189	[1, 3, 5]	212	[4, 7, 5]
190	[1, 6, 5, 1]	213	[5, 2, 3]
191	[7, 1, 3, 7]	214	[1, 2, 1, 4]
192	[6, 4, 7, 3]	215	[2, 5, 3]
193	[2, 5, 1, 3]	216	[7, 4, 2, 5]
194	[7, 6, 2]	217	[2, 3, 2, 5]
195	[6, 7, 2, 6]	218	[1, 7, 3]
196	[6, 1, 5]	219	[7, 5, 2, 4]
197	[1, 2, 7, 4]	220	[1, 5, 7, 6]
198	[4, 1, 6, 2]	221	[2, 7, 6, 2]
199	[4, 2, 6, 1]	222	[3, 2, 4, 5]
200	[6, 4, 1, 3]	223	[7, 5, 6, 4]
201	[3, 2, 5]	224	[7, 6, 5, 2]

225	[5, 2, 4, 3]	233	[1, 5, 2, 6, 1]
226	[7, 4, 6, 5]	234	[1, 3, 5, 7, 1]
227	[7, 1, 2, 3]	235	[5, 6, 7, 1, 5]
228	[1, 4, 7, 2]	236	[2, 4, 6, 1, 2]
229	[7, 5, 1, 4]	237	[1, 3, 4, 7, 1]
230	[5, 7, 4]	238	[7, 1, 2, 3, 7]
231	[7, 1, 6, 3]	239	[1, 2, 3, 4, 1]
232	[3, 6, 4]	240	[3, 7, 4, 1, 3]

Define (t_i):

(t_0):

(1, 2)(3, 6)(4, 7)(5, 10)(8, 16)(9, 17)(11, 22)(12, 24)(13, 27)(14, 29)(15, 31)(18, 35)(19, 36)(20, 40)(21, 41)(23, 45)(25, 48)(26, 51)(28, 54)(30, 57)(32, 61)(33, 63)(34, 65)(37, 52)(38, 73)(39, 74)(42, 81)(43, 83)(44, 85)(46, 88)(47, 89)(49, 92)(50, 94)(53, 79)(55, 72)(56, 99)(58, 103)(59, 104)(60, 105)(62, 106)(64, 107)(66, 82)(67, 111)(68, 113)(69, 75)(70, 116)(71, 87)(76, 120)(77, 122)(78, 124)(80, 126)(84, 130)(86, 134)(90, 136)(91, 137)(93, 141)(95, 143)(96, 144)(97, 145)(98, 146)(100, 147)(101, 148)(102, 150)(108, 157)(109, 159)(110, 162)(112, 165)(114, 168)(115, 171)(117, 173)(118, 174)(119, 175)(121, 177)(123, 180)(125, 182)(127, 186)(128, 188)(129, 190)(131, 192)(132, 193)(133, 195)(135, 197)(138, 199)(139, 200)(140, 198)(142, 202)(149, 176)(151, 210)(152, 203)(153, 211)(154, 172)(155, 163)(156, 204)(158, 179)(160, 181)(161, 206)(164, 191)(166, 218)(167, 187)(169, 205)(170, 220)(178, 201)(183, 208)(184, 215)(185, 196)(189, 228)(194, 221)(207, 209)(212, 214)(213, 225)(216, 233)(217, 230)(219, 234)(222, 232)(223, 235)(224, 236)(226, 237)(227, 238)(229, 239)(231, 240)

(t_1):

(1, 3)(2, 13)(4, 21)(5, 11)(6, 27)(7, 55)(8, 59)(9, 18)(10, 69)(12, 90)(14, 101)(15, 33)(16, 105)(17, 108)(19, 87)(20, 42)(22, 75)(23, 78)(24, 83)(25, 139)(26, 52)(28, 67)(29, 146)(30, 138)(31, 61)(32, 63)(34, 128)(35, 157)(36, 143)(37, 97)(38, 76)(39, 80)(40, 178)(41, 72)(43, 136)(44, 86)(45, 66)(46, 166)(47, 132)(48, 173)(49, 118)(50, 77)(51, 145)(53, 96)(54, 99)(56, 111)(57, 182)(58, 142)(60, 104)(62, 100)(64, 152)(65, 175)(68, 210)(70, 216)(71, 95)(73, 183)(74, 222)(79, 225)(81, 201)(82, 124)(84, 229)(85, 191)(88, 154)(89, 211)(91, 174)(92, 137)(93, 121)(94, 184)(98, 148)(102, 202)(103, 150)(106, 176)(107, 192)(109, 171)(110, 163)(112, 207)(113, 186)(114, 190)(115, 223)(116, 158)(117, 200)(119, 188)(120, 208)(122, 215)(123, 181)(125, 199)(126, 232)(127, 151)(129, 224)(130, 167)(131, 203)(133, 226)(134, 164)(135, 205)(140, 204)(141, 206)(144, 213)(147, 149)(153, 193)(155, 217)(156, 214)(159, 235)(160, 231)(161, 177)(162, 230)(165, 196)(168, 236)(169, 227)(170, 195)(172, 218)(179, 233)(180, 240)(185, 209)(187, 239)(189, 221)(194, 219)(197, 238)(198, 212)(220, 237)(228, 234)

(t_2):

(1, 5)(2, 23)(3, 26)(4, 64)(6, 82)(7, 112)(8, 60)(9, 20)(10, 45)(11, 52)(12, 194)(13, 97)(14, 91)(15, 28)(16, 104)(17, 103)(18, 80)(19, 135)(21, 121)(22, 124)(24, 113)(25, 117)(27, 75)(29, 163)(30, 125)(31, 73)(32, 56)(33, 156)(34, 86)(35, 202)(36, 159)(37, 78)(38, 54)(39, 42)(40, 58)(41, 185)(43, 228)(44, 123)(46, 47)(48, 200)(49, 98)(50, 87)(51, 66)(53, 110)(55, 206)(57, 199)(59, 105)(61, 198)(62, 149)(63, 120)(65, 195)(67, 214)(68, 221)(69, 145)(70, 219)(71, 115)(72, 131)(74, 150)(76, 204)(77, 205)(79, 174)(81, 102)(83, 158)(84, 224)(85, 220)(88, 129)(89, 190)(90, 216)(92, 144)(93, 152)(94, 171)(95, 227)(96, 146)(99, 140)(100, 176)(101, 225)(106, 147)(107, 165)(108, 222)(109, 197)(111, 208)(114, 187)(116, 186)(118, 162)(119, 191)(122, 235)(126, 142)(127, 234)(128, 181)(130, 132)(133, 134)(136, 151)(137, 155)(138, 182)(139, 173)(141, 207)(143, 215)(148, 230)(153, 172)(154, 229)(157, 201)(160, 226)(161, 192)(164, 231)(166, 167)(168, 218)(169, 223)(170, 180)(175, 240)(177, 196)(178, 232)(179, 189)(183, 212)(184, 238)(188, 237)(193, 236)(203, 209)(210, 233)(211, 239)(213, 217)

(t_3):

(1, 9)(2, 88)(3, 44)(4, 93)(5, 50)(6, 218)(7, 48)(8, 53)(10, 89)(11, 128)(12, 70)(13, 164)(14, 79)(15, 38)(16, 29)(17, 46)(18, 86)(19, 123)(20, 87)(21, 152)(22, 193)(23, 115)(24, 62)(25, 141)(26, 135)(27, 157)(28, 73)(30, 76)(31, 54)(32, 208)(33, 138)(34, 77)(35, 172)(36, 168)(37, 84)(39, 52)(40, 190)(41, 200)(42, 181)(43, 179)(45, 58)(47, 94)(49, 96)(51, 187)(55, 192)(56, 120)(57, 99)(59, 118)(60, 213)(61, 182)(63, 111)(64, 121)(65, 211)(66, 170)(67, 199)(68, 127)(69, 175)(71, 129)(72, 161)(74, 130)(75, 215)(78, 226)(80, 205)(81, 236)(82, 159)(83, 189)(85, 166)(90, 219)(91, 225)(92, 146)(95, 231)(97, 227)(98, 144)(100, 221)(101, 174)(102, 124)(103, 171)(104, 148)(105, 137)(106, 116)(107, 173)(108, 191)(109, 220)(110, 217)(112, 185)(113, 151)(114, 180)(117, 177)(119, 184)(122, 153)(125, 183)(126, 239)(131, 206)(132, 188)(133, 223)(134, 154)(136, 149)(139, 203)(140, 214)(142, 195)(143, 201)(145, 232)(147, 233)(150, 237)(155, 162)(156, 198)(158, 228)(160, 224)(163, 230)(165, 209)(167, 197)(169, 229)(176, 234)(178, 240)(186, 210)(194, 216)(196, 207)(202, 235)(204, 212)(222, 238)

(t_4):

(1, 15)(2, 71)(3, 83)(4, 72)(5, 47)(6, 36)(7, 41)(8, 49)(9, 58)(10, 94)(11, 186)(12, 95)(13, 90)(14, 98)(16, 230)(17, 40)(18, 100)(19, 43)(20, 103)(21, 55)(22, 122)(23, 129)(24, 33)(25, 131)(26, 167)(27, 32)(28, 46)(29, 148)(30, 86)(31, 87)(34, 119)(35, 81)(37, 169)(38, 45)(39, 232)(42, 147)(44, 170)(48, 185)(50, 89)(51, 197)(52, 214)(53, 118)(54, 171)(56, 172)(57, 181)(59, 96)(60, 91)(61, 136)(62, 142)(63, 143)(64, 117)(65, 188)(66, 189)(67, 116)(68, 184)(69, 210)(70, 223)(73, 190)(74, 126)(75, 153)(76, 221)(77, 127)(78, 219)(79, 162)(80, 222)(82, 168)(84, 227)(85, 180)(88, 115)(92, 217)(93, 209)(97, 229)(99, 233)(101, 146)(102, 157)(104, 163)(105, 225)(106, 178)(107, 177)(108, 176)(109, 179)(110, 174)(111, 235)(112, 200)(113, 132)(114, 228)(120, 236)(121, 173)(123, 220)(124, 208)(125, 191)(128, 175)(130, 156)(133, 231)(134, 160)(135, 187)(137, 213)(138, 195)(139, 206)(140, 145)(141, 203)(144, 155)(149, 150)(151, 211)(152, 207)(154, 216)(158, 166)(159, 218)(161, 165)(164, 226)(182, 237)(183, 234)(192, 196)(193, 215)(194, 224)(198, 239)(199, 240)(201, 202)(204, 238)(205, 212)

(t_5):

(1, 8)(2, 37)(3, 66)(4, 25)(5, 75)(6, 51)(7, 141)(9, 62)(10, 22)(11, 69)(12, 84)(13, 78)(14, 46)(15, 30)(16, 52)(17, 74)(18, 142)(19, 114)(20, 201)(21, 196)(23, 97)(24, 53)(26, 82)(27, 60)(28, 56)(29, 116)(31, 212)(32, 125)(33, 76)(34, 153)(35, 126)(36, 180)(38, 138)(39, 106)(40, 81)(41, 177)(42, 178)(43, 187)(44, 189)(45, 59)(47, 119)(48, 93)(49, 86)(50, 151)(54, 111)(55, 207)(57, 214)(58, 100)(61, 183)(63, 204)(64, 131)(65, 122)(67, 99)(68, 193)(70, 88)(71, 160)(72, 117)(73, 198)(77, 211)(79, 130)(80, 202)(83, 170)(85, 197)(87, 217)(89, 127)(90, 226)(91, 191)(92, 205)(94, 188)(95, 224)(96, 195)(98, 172)(101, 171)(102, 176)(103, 222)(104, 145)(105, 124)(107, 203)(108, 150)(109, 166)(110, 143)(112, 206)(113, 215)(115, 216)(118, 221)(120, 156)(121, 185)(123, 168)(128, 210)(129, 231)(132, 184)(133, 229)(134, 169)(135, 228)(136, 213)(137, 234)(139, 165)(140, 199)(144, 239)(146, 235)(147, 232)(148, 233)(149, 157)(152, 192)(154, 223)(155, 190)(158, 159)(161, 200)(162, 236)(163, 240)(164, 219)(167, 220)(173, 209)(174, 238)(175, 186)(179, 218)(181, 230)(182, 208)(194, 227)(225, 237)

(t_6):

(1, 4)(2, 12)(3, 19)(5, 34)(6, 43)(7, 24)(8, 14)(9, 39)(10, 68)(11, 77)(13, 95)(15, 32)(16, 79)(17, 106)(18, 42)(20, 80)(21, 87)(22, 127)(23, 133)(25, 46)(26, 123)(27, 72)(28, 125)(29, 53)(30, 56)(31, 63)(33, 61)(35, 147)(36, 83)(37, 70)(38, 140)(40, 176)(41, 136)(44, 135)(45, 165)(47, 153)(48, 130)(49, 91)(50, 128)(51, 179)(52, 93)(54, 204)(55, 143)(57, 120)(58, 102)(59, 155)(60, 98)(62, 74)(64, 86)(65, 113)(66, 109)(67, 183)(69, 184)(71, 90)(73, 199)(75, 119)(76, 99)(78, 223)(81, 100)(82, 220)(84, 88)(85, 228)(89, 193)(92, 174)(94, 210)(96, 163)(97, 231)(101, 217)(103, 202)(104, 144)(105, 162)(107, 221)(108, 178)(110, 146)(111, 212)(112, 195)(114, 166)(115, 226)(116, 141)(117, 172)(118, 137)(121, 181)(122, 186)(124, 209)(126, 149)(129, 229)(131, 191)(132, 211)(134, 194)(138, 198)(139, 190)(142, 150)(145, 161)(148, 213)(151, 188)(152, 205)(154, 224)(156, 182)(157, 232)(158, 180)(159, 170)(160, 216)(164, 227)(167, 168)(169, 219)(171, 196)(173, 236)(175, 215)(177, 233)(185, 237)(187, 218)(189, 197)(192, 238)(200, 239)(201, 222)(203, 234)(206, 240)(207, 235)(208, 214)(225, 230)

The group is defined by the symmetric presentation. It index is at most:

$$\frac{|N|}{|N|}+\frac{|N|}{|N^{(0)}|}+\frac{|N|}{|N^{(01)}|}+\frac{|N|}{|N^{(010)}|}+\frac{|N|}{|N^{(012)}|}+\frac{|N|}{|N^{(013)}|}+\frac{|N|}{|N^{(0103)}|}+\frac{|N|}{|N^{(0130)}|}+\frac{|N|}{|N^{(0123)}|}+\frac{|N|}{|N^{(01320)}|}$$

$= 1 + 7 + 42 + 7 + 56 + 42 + 7 + 14 + 56 + 7 = 240$

(According to the S_8 graph)

The index of N in G has most 240. $\Rightarrow$ Order of G is at most $240|N| = 240 \times 168 = 40320$. (At this point G has order at most 40320. Since $|N|$ is 168)

Thus, the order of G is most at 40320.

Since the group S_8 is generated by x, y, and t

$\Rightarrow$ S_8 is an image of G.

$\Rightarrow |G| \geq |S_8| \Rightarrow |G| \geq 40320$

$\Rightarrow 40320 \leq |G| \leq 40320 \Rightarrow |G| = 40320$

$\Rightarrow G \cong S_8$

CHAPTER FOUR

SYMMETRIC GENERATION OF $3^{\cdot}S_7$

Symmetric Presentation

The same symmetric presentations $2^{*7}: L_3(2)$ are given by $< x,y,t \mid x^7 = y^2 = (xy)^3 = [x,y]^4 = 1 = t^2 = [xy, t^{x^4}] = [y, t^{x^3}] >$, where $L_3(2) = <x,y>$, and the action of N on the symmetric generators is given by $x \sim (0,1,2,3,4,5,6)$, $y \sim (2,6)(4,5)$. We factor by the relation $[(0\ 1\ 2\ 3\ 4\ 5\ 6)\ t_0]^6$ to obtain

$$G = \frac{2^{*7}: L_3(2)}{[(0123456)t_0]^6} \cong 3^{\cdot}S_7 \text{ (see [4])}.$$

Let $N = L_3(2)$. Now N is transitive on the set of symmetric generators $\omega = \{0,1,..,6\}$. The double coset Nt_iN contains 7 single cosets. $N^0 = \langle (1,2,3,4,5,6,7), (2,6)(4,5) \rangle$ is transitive on $\{1,2,3,4,5,6\}$, and its orbits on ω are $\{0\}$, $\{1,2,3,4,5,6\}$.

The relation $[\pi t_0]^6 = 1$, where $\pi = (0\ 1\ 2\ 3\ 4\ 5\ 6)$ gives

$\pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0\ \pi\ t_0 = \pi^6\ \pi^{-5}\ t_0\ \pi^5\ \pi^{-4}\ t_0\ \pi^4\ \pi^{-3}\ t_0\ \pi^3\ \pi^{-2}\ t_0\ \pi^2\ \pi^{-1}\ t_0\ \pi\ t_0$

$\Rightarrow [(0\ 1\ 2\ 3\ 4\ 5\ 6)\ t_0]^6 = [\pi\ t_0]^6 = \pi^6\ t_5\ t_4\ t_3\ t_2\ t_1\ t_0$

$= (1\ 0\ 6\ 5\ 4\ 3\ 2)\ t_5\ t_4\ t_3\ t_2\ t_1\ t_0$

Similarly, $[(1\ 2\ 4)(3\ 6\ 5)\ t_1]^6 = t_4\ t_2\ t_1\ t_4\ t_2\ t_1$,

$[(1\ 2\ 4)(3\ 6\ 5)\ t_3]^{10} = (1\ 2\ 4)(3\ 6\ 5)\ t_3\ t_5\ t_6\ t_3\ t_5\ t_6\ t_3\ t_5\ t_6\ t_3$

$[(2\ 6)(4\ 5)\ t_2]^{12} = t_6\ t_2\ t_6\ t_2\ t_6\ t_2\ t_6\ t_2\ t_6\ t_2\ t_6\ t_2$,

$[(1\ 3)(2\ 4\ 6\ 5)\ t_1]^6 = (2\ 6)(4\ 5)\ t_3\ t_1\ t_3\ t_1\ t_3\ t_1$,

$[(1\ 3)(2\ 4\ 6\ 5)\ t_2]^{12} = t_5\ t_6\ t_4\ t_2\ t_5\ t_6\ t_4\ t_2\ t_5\ t_6\ t_4\ t_2$,

$[(2\ 6)(4\ 5)\ t_0\ t_2]^{12} = t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2$

Since $e = (1765432)\ 543\underline{217} \Rightarrow 712 = (1765432)\ 543$

$712^{(1573264)} = (1765432)\ 543^{(1573264)} \Rightarrow \underline{356} = (1265347)\ \underline{712}$

$\Rightarrow 356\ \underline{(1743562)} = \underline{(1265347)\ 712}\ (1743562)$

$\Rightarrow (1743562)\ 562 = 471$

$\Rightarrow (1743562)\ 562^{(1265347)} = 471^{(1265347)}$

$\Rightarrow (1743562)\ 356 = 712$

Since $712 = (1765432)\ 543$ and $712 = (1743562)\ 356$

$\Rightarrow \underline{(1765432)}\ 543 = (1743562)\ 356$

$\Rightarrow \underline{(1765432)\ 543}\ (1234567) = \underline{(1743562)\ 356}\ (1234567)$

$\Rightarrow 654 = (36)(57)\ 467 \Rightarrow 654^{(1367425)} = (36)(57)\ 467^{(1367425)}$

$\Rightarrow 712 = (14)(67)\ 274$

Since $712 = (14)(67)\ 274$ and $712 = (1765432)\ 543$

$\Rightarrow \underline{(14)(67)}\ 274 = (1765432)\ 543$

$\Rightarrow \underline{(14)(67)\ 274}\ (14)(67) = \underline{(1765432)\ 543}\ (14)(67)$

$\Rightarrow 261 = (165)(243)\ 513$

$\Rightarrow 261^{(1273546)} = (165)(243)\ 513^{(1273546)}$

$\Rightarrow 712 = (142)(576)\ 425$

Since $712 = (14)(67)\ 274$ and $712 = (142)(576)\ 425$

$\Rightarrow (14)(67)\ 274 = \underline{(142)(576)}\ 425$

$\Rightarrow \underline{(14)(67)\ 274}\ (124)(567) = \underline{(142)(576)\ 425}\ (124)(567)$

$\Rightarrow (24)(56)\ 451 = 146$

$\Rightarrow (24)(56)\ 451^{(1743562)} = 146^{(1743562)}$

$\Rightarrow (13)(26)\ 367 = 732 \Rightarrow (13)(26)\ 367^{(13)(45)} = 732^{(13)(45)}$

$\Rightarrow (13)(26)\ 167 = 712$

Since $712 = (1765432)\ 543$ and $712 = (14)(67)\ 274$

$\Rightarrow \underline{(14)(67)}\ 274 = (1765432)\ 543$

$\Rightarrow \underline{(14)(67)\ 274}\ (14)(67) = \underline{(1765432)\ 543}\ (14)(67)$

$\Rightarrow 261 = \underline{(165)(243)}\ 513$

$\Rightarrow 261\ (156)(234) = \underline{(165)(243)\ 513}\ (156)(234)$

$\Rightarrow (156)(234)\ 315 = 654$

$\Rightarrow (156)(234)\ 315^{(1367425)} = 654^{(1367425)}$

$\Rightarrow (173)(256)\ 631 = 712$

Thus,

$712 = (1765432)\ 543 = (1743562)\ 356 = (14)(67)\ 274$

$= (142)(576)\ 425 = (13)(26)\ 167 = (173)(256)\ 631$

$\Rightarrow 712 \sim 543 \sim 356 \sim 274 \sim 425 \sim 167 \sim 631$

Since $e = (26)(45)\ 313131 \Rightarrow e = (26)(45)\ 313131^{(173)(245)}$

$\Rightarrow e = (25)(46)\ 171\underline{717} \Rightarrow 717 = (25)(46)\ 171$

Since $71\underline{2} = (142)(576)\ 425$ and $71 = (25)(46)\ 1717$

$\Rightarrow \underline{(25)(46)}\ 1717 = 71 = (142)(576)\ 4252$

$\Rightarrow \underline{(25)(46)\ 1717}\ (25)(46) = (142)(576)\ \underline{4}252\ \underline{(25)(46)}$

$\Rightarrow 1717 = 1\ (162)(457)\ 525$

Since $1 = 1 \Rightarrow \underline{11}717 = \underline{11}\ (162)(574)\ 525$

$\Rightarrow 717 = (162)(457)\ 525$

Since $e = (17)(46)\ 252\underline{525}$

$\Rightarrow 525 = (17)(46)\ 252$ and $717 = (162)(457)\ 525$

$\Rightarrow 717 = \underline{(162)(457)(17)(46)}\ 252 = (145)(276)\ 252$

Since $717 = (162)(457)\ 525$

$\Rightarrow 717^{(26)(45)} = (162)(457)\ 525^{(26)(45)}$

$\Rightarrow 717 = (126)(475)\ 464$

Since $717 = (126)(475)\ 464$ and $e = \underline{(17)(25)}\ 646464$

$\Rightarrow 717 = \underline{(126)(475)(17)(25)}\ 646 \quad \Rightarrow 717 = (154)(267)\ 646$

Thus,

$717 = (25)(46)\ 171 = (162)(457)\ 525 = (145)(276)\ 252$

$= (126)(475)\ 464 = (154)(267)\ 646$

$\Rightarrow 717 \sim 171 \sim 525 \sim 252 \sim 464 \sim 646$

Since $717 = (126)(475)\ 464$ and $e = 643\underline{643}$

$\Rightarrow 7173 = (126)(475)\ 4643$ and $643 = 346$

$\Rightarrow$ 7173 = (126)(475) 4346

Since e = 7173$\underline{3717}$ $\Rightarrow$ 7173 = 7173

$\Rightarrow$ 7173 = 7371 [Since e = 371$\underline{371}$]

Since 7173 = (126)(475) 4643

$\Rightarrow$ $7173^{(13)(26)}$ = (126)(475) $4643^{(13)(26)}$

$\Rightarrow$ 7371 = (236)(475) 4241

$\Rightarrow$ 7371 = (236)(475) 4142 [Since e = 421$\underline{421}$]

Since 7173 = (126)(475) $\underline{434}$6

$\Rightarrow$ 7173 = (126)(475)(15)(27) 3436

[Since e = (15)(27) 343$\underline{434}$]

$\Rightarrow$ 7173 = (17)(2654) 3436

$\Rightarrow$ 7173 = (17)(2654) 3634 [Since e = 346$\underline{346}$]

Since 7173 = $\underline{717}$3

$\Rightarrow$ 7173 = (25)(46) 1713 [Since e = (25)(46)717$\underline{171}$]

$\Rightarrow$ 7173 = (25)(46) 1317 [Since e = 317$\underline{317}$]

$\Rightarrow$ 7173 = (25)(46)(26)(45) 3137 [Since e = (26)(45) 313$\underline{131}$]

$\Rightarrow$ 7173 = (24)(56) 3137

Since 7173 = (126)(475) 4346 and 7173 = (236)(475) 4142

$\Rightarrow$ $\underline{(236)(475)}$ 4142 = (126)(475) 4346

$\Rightarrow$ $\underline{(236)(475)\ 4142}$ (263)(457) = $\underline{(126)(475)\ 4346}$ (263)(457)

$\Rightarrow$ 5156 = (16)(23) 5253

$$\Rightarrow 5156^{(1357246)} = (16)(23)\ 5253^{(1357246)}$$

$$\Rightarrow 7371 = (13)(45)\ 7475$$

Thus,

$$7173 = (126)(475)\ 4346 = 7371 = (236)(475)\ 4142$$

$$= (17)(2654)\ 3634 = (24)(56)\ 3137 = (13)(45)\ 7475$$

$$\Rightarrow 7173 \sim 4346 \sim 7371 \sim 4142 \sim 3634 \sim 3137 \sim 7475$$

Since $e = 713\underline{713} \Rightarrow 713 = 317$

Since $e = \underline{(25)(46)}\ 171717 = (1265347)(1765432)\ 171717$

$$= (1265347)\underline{(1765432)\ 1717176}6$$

$$= (1265347)\ 2121\underline{217\ (1765432)}\ 6$$

$$= \underline{(1265347)}\ 21213\underline{45}6 = (1637254)\underline{(1743562)\ 2121343}356$$

$$= (1637254)\ 6262474\ \underline{(1743562)\ 356} = \underline{(1637254)}\ 6262474712$$

$$= (1674)(25)\underline{(24)(37)\ 6262474712}$$

$$= (1674)(25)\ 6464232314\ \underline{(24)(37)}$$

$$\Rightarrow (24)(37) = \underline{(1674)(25)}\ 6464232314$$

$$= (25)(46)\underline{(16)(47)\ 6464}232314$$

$$= \underline{(25)(46)\ 1717}\ \underline{(16)(47)\ 2323}14 = 713214$$

[Since $e = (25)(46)\ 1717\underline{17}$ and $e = (16)(47)\ 2323\underline{23}$]

$$\Rightarrow (24)(37) = 713\underline{214} \Rightarrow (24)(37)\ 412 = 713$$

Since $713 = (24)(37)\ 412 \Rightarrow 713^{(24)(56)} = (24)(37)\ 412^{(24)(56)}$

$$\Rightarrow 713 = (24)(37)\ 214$$

Since $713 = (24)(37)\ 412 \Rightarrow 713^{(25)(46)} = (24)(37)\ 412^{(25)(46)}$

$\Rightarrow 713 = (37)(56)\ 516$

Since $713 = (24)(37)\ 412 \Rightarrow 713^{(26)(45)} = (24)(37)\ 412^{(26)(45)}$

$\Rightarrow 713 = (37)(56)\ 615$

Thus,

$713 = 317 = (24)(37)\ 214 = (24)(37)\ 412$

$= (37)(56)\ 516 = (37)(56)\ 615$

$\Rightarrow 713 \sim 317 \sim 214 \sim 412 \sim 516 \sim 615$

Since $e = 713\underline{713}$ $\Rightarrow 7131 = 3\underline{171}$

$\Rightarrow 7131 = \underline{3\ (25)(46)}\ 717$ [Since $e = (25)(46)\ 717\underline{171}$]

$\Rightarrow 7131 = (25)(46)\ \underline{371}7$

$\Rightarrow 7131 = (25)(46)\ 1\underline{737}$ [Since $e = 173\underline{173}$]

$\Rightarrow 7131 = \underline{(25)(46)\ 1\ (24)(56)}\ 373$

[Since $e = (24)(56)\ 373\underline{737}$]

$\Rightarrow 7131 = (26)(45)\ 1373$

Since $713 = (37)(56)\ 615 \Rightarrow 7131 = (37)(56)\ 6\underline{151}$

$\Rightarrow 7131 = \underline{(37)(56)\ 6\ (27)(34)}\ 515$

[Since $e = (27)(34)\ 515\underline{151}$]

$\Rightarrow 7131 = (2743)(56)\ 6515$

Since $7131 = (37)(56)\ 6151$

$\Rightarrow 7131 = (37)(56)\ 5\underline{161}$ [Since $e = 516\underline{516}$]

$\Rightarrow 7131 = \underline{(37)(56)\ 5\ (23)(47)}\ 616$

$[\text{Since } e = (23)(47)\ 616\underline{161}]$

$\Rightarrow 7131 = (2347)(56)\ 5616$

$\text{Since } 71\underline{3} = (24)(37)\ 21\underline{4} \Rightarrow 7131 = (24)(37)\ 2\underline{141}$

$\Rightarrow 7131 = \underline{(24)(37)\ 2\ (35)(67)}\ 414 \Rightarrow 7131 = (24)(3675)\ \underline{241}4$

$\Rightarrow 7131 = (24)(3675)\ 1424\ [\text{Since } e = 421\underline{421}]$

$\text{Since } e = 71\underline{37}13 \Rightarrow e = 7131\underline{1713} \Rightarrow 7131 = 3171$

$[\text{Since } e = (25)(46)\ 717\underline{171}] \Rightarrow 7131 = \underline{3\ (25)(46)}\ 717$

$\Rightarrow 7131 = (25)(46)\ 3\underline{717} \Rightarrow 7131 = \underline{(25)(46)\ 3\ (145)(276)}\ 252$

$[\text{Since } 717 = (145)(276)\ 252] \Rightarrow 7131 = (142)(576)\ \underline{325}2$

$\Rightarrow 7131 = (142)(576)\ 5232\ [\text{Since } e = 523\underline{523}]$

$\text{Since } 7131 = (142)(576)\ 5232 \text{ and } 7131 = (24)(3675)\ 1424$

$\Rightarrow (142)(576)\ 5232 = \underline{(24)(3675)}\ 1424$

$\Rightarrow \underline{(142)(576)\ 5232}\ (24)(3576) = \underline{(24)(3675)\ 1424}\ (24)(3576)$

$\Rightarrow (12)(3567)\ 7454 = \underline{1242}$

$\Rightarrow (12)(3567)\ 7454 = \underline{(24)(3675)}\ 7131$

$[\text{Since } 7131 = (24)(3576)\ 1242]$

$\Rightarrow \underline{(12)(3567)\ 7454}\ (24)(3576) = \underline{(24)(3675)\ 7131}\ (24)(3576)$

$\Rightarrow (142)(375)\ 6272 = 6151$

$\Rightarrow (142)(375)\ 6272^{(24)(3675)} = 6151^{(24)(3675)}$

$\Rightarrow 7131 = (124)(365)\ \underline{745}4$

$\Rightarrow 7131 = (124)(365)\ 5\underline{474}$ [Since $e = 547\underline{547}$]

$\Rightarrow 7131 = \underline{(124)(365)\ 5\ (16)(23)}\ 747$

[Since $e = (16)(23)\ 747\underline{474}$]

$\Rightarrow 7131 = (13)(2465)\ 5747$

Thus,

$7131 = (13)(2465)\ 5747 = (26)(45)\ 1373$

$= (2743)(56)\ 6515 = (2347)(56)\ 5616$

$= (24)(3675)\ 1424 = (142)(576)\ 5232$

$\Rightarrow 7131 \sim 5747 \sim 1373 \sim 6515 \sim 5616 \sim 1424 \sim 5232$

Summarize:

The set of all double cosets $[\omega] = N\omega N$, coset stabilizing subgroups $N^{(\omega)}$, and the number of single cosets in each double coset are displayed below.

Table 4.1. The Double Coset $N\omega N = [\omega]$, where $N = L_3(2)$, $\omega = \{0,1,2,3,4,5,6\}$

$[\omega]$	Coset Stabilising subgroup $N^{(\omega)}$	Number of cosets in $[\omega]$
[*]	N is transitive on $\omega = \{0,1,2,3,4,5,6\}$	1
[0]	$N^{(0)} = N^0 \cong \langle(2,5)(4,6),\ (2,6)(4,5),$ $(1,5,6)(2,3,4)\rangle$, $\lvert N^{(0)}\rvert = 24$, $N^{(0)}$ has orbits $\{0\}$ and $\{1,2,3,4,5,6\}$ on	7

	ω	
[01]	$N^{01} \cong \langle (2,6)(4,5),\ (2,4)(5,6) \rangle$, $\lvert N^{(01)} \rvert = 4$, $N^{(0)}$ has orbits {1}, {3}, {0}, and {2,4,5,6} on ω	42
[010]	$N^{010} \cong \langle (2,6)(4,5),\ (2,4)(5,6) \rangle$. Now 717 ~ 464 ~ 171 ~ 525 ~ 616 ~252 $\Rightarrow N^{(010)} \cong \langle N^{010}$, (1,4,2)(5,7,6), (1,6,7,4)(2,5), (1,7)(2,6,5,4), (1,4)(6,7), (1,7)(2,5), (1,2,7,5)(4,6), (1,5,7,2)(4,6), (1,6,2)(4,5,7), (1,2,4)(5,6,7), (1,7)(2,4,5,6), (1,4,7,6)(2,5), (1,2)(5,7), (1,7)(4,6), (1,5,6)(2,4,7), (1,6,5)(2,7,4), (1,5)(2,7), (1,6)(4,7), (2,4)(5,6), (1,2,6)(4,7,5), (1,4,5)(2,7,6), (2,6)(4,5), (1,5,4)(2,6,7)$\rangle$ Now $\lvert N^{(010)} \rvert = 24$ and $N^{(010)}$ has orbits {3}, and {0,1,2,4,5,6}	7
[012]	$N^{012} = \langle e \rangle$. But 712 ~ 631 ~ 543 ~ 274 ~ 425 ~ 356 ~ 167. Thus, $N^{(012)} \cong \langle$(1,5,7,3,2,6,4), (1,3,4,7,6,5,2), (1,2,5,6,7,4,3), (1,7,2,4,5,3,6), (1,4,6,2,3,7,5), (1,6,3,5,4,2,7)$\rangle$, $\lvert N^{(012)} \rvert = 7$ and $N^{(012)}$ is transitive on ω	24
[013]	$N^{013} \cong \langle (2,5)(4,6),\ (2,6)(4,5) \rangle$. Now 713 ~ 615 ~ 412 ~ 516 ~ 214 ~ 317 $\Rightarrow N^{(013)} \cong \langle N^{013}$, (3,6)(5,7), (3,7)(5,6), (2,3,4,7)(5,6), (2,6,4,5)(3,7), (2,5,4,6)(3,7), (2,3,5)(4,7,6), (2,4)(3,7), (2,5)(4,6), (2,4)(3,5,7,6), (2,7,6)(3,5,4), (2,5,7)(3,4,6), (2,6,3)(4,5,7), (2,3)(4,7), (2,5,3)(4,6,7), (2,7,5)(3,6,4), (2,7)(3,4), (2,6)(4,5), (2,3,6)(4,7,5), (2,6,7)(3,4,5), (3,5)(6,7), (2,7,4,3)(5,6), (2,4)(3,6,7,5)$\rangle$, Now $\lvert N^{(013)} \rvert = 24$ and $N^{(013)}$ has orbits {1}	7

	and {2,3,4,5,6,0}	
[0103]	N^{0103} = <e>. But 7173 ~ 7371 ~ 7672 ~ 2124 ~ 7475 ~ 7276 ~ 1313. Thus, $N^{(0103)} \cong$ <(1,2,5)(3,6,4), (1,4,6)(2,3,5), (1,2,3,6)(4,5), (1,3)(2,4,6,5), (1,3)(4,5), (1,4,2)(3,5,6), (1,6)(2,3), (1,3)(2,5,6,4), (1,3,7)(2,6,5), (1,6,3,2)(4,5), (1,3,7)(2,4,6), (1,2)(3,6), (1,3)(2,6), (2,7)(3,4), (2,6,7)(3,4,5), (1,4)(3,5), (1,2,4)(3,6,5), (1,3,7)(4,5,6), (1,6,4)(2,5,3), (2,5,7)(3,4,6), (2,4)(5,6), (1,3,7)(2,5,4), (2,3,4,7)(5,6), (1,6,5)(2,4,3), (1,4,3,5)(2,6), (2,6)(4,5)> Now $\lvert N^{(0103)}\rvert$ = 168 and $N^{(0103)}$ is transitive on ω	1
[0131]	N^{0131} = <e>. But 7131 ~ 5232 ~ 1373 ~ 1424 ~ 6515 ~ 5616 ~ 5747 Thus, $N^{(0131)} \cong$ <(1,5,7,6,4,2,3), (1,3,7)(2,5,4), (1,7,5)(3,4,6), (1,7,5,2)(3,4), (1,4,7)(2,5,3), (2,6)(4,5), (1,2,7,5)(4,6), (1,4,6,7)(2,3), (2,4)(5,6), (1,2,6)(4,7,5), (1,2)(5,7), (1,6,3)(4,7,5), (1,7,5,3,4,2,6), (1,6,7,5,2,4,3), (1,4,5,6,3,2,7), (1,5,2,4,7,6,3), (1,7,5,6,2,3,4), (1,2,4)(5,6,7), (1,4,3,2,6,5,7), (1,5,4,3)(6,7), (1,6,2,3)(5,7), (1,6,4,2,7,5,3), (1,3,7)(2,6,5), (1,5,3)(2,7,6), (1,3,7)(2,4,6), (1,3,7)(4,5,6)> $\lvert N^{(0131)}\rvert$ = 168 and $N^{(0131)}$ is transitive on ω	1

It is readily verified that

$3^{\cdot}S_7 \cong <t_0t_1t_2t_3t_4t_5t_6,\ t_0> = G$

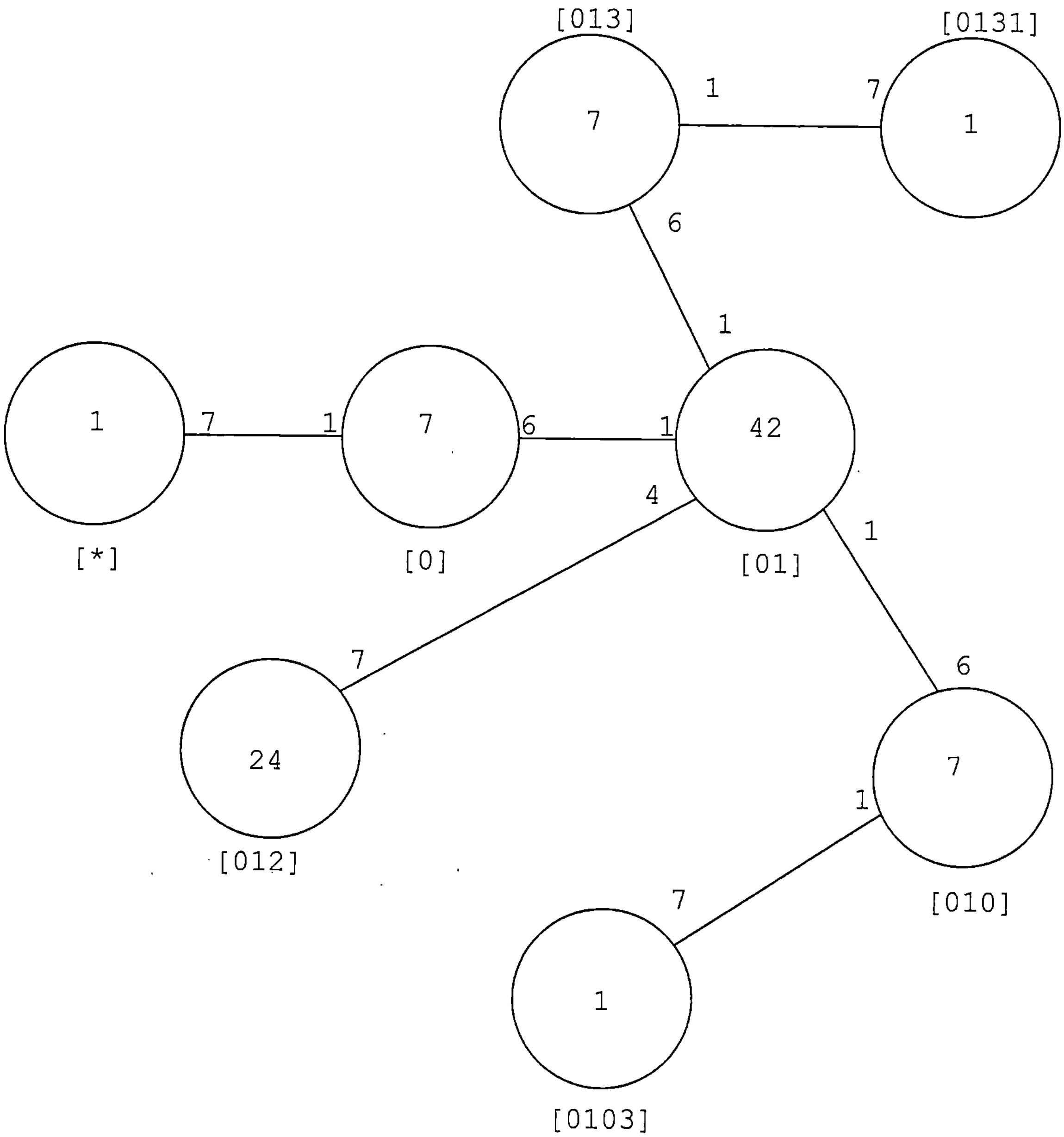

Figure 4.1. Cayley Graph of $3^{\cdot}S_7$ over $L_3(2)$

The Identification of Cosets Labeling

MAGMA Labeling-Single coset

1 [*]

2 [7]

3 [1]

4 [6]

5 [2]

6 [1, 7]

7 [6, 7]

8 [5]

9 [3]

10 [2, 7]

11 [2, 1]

12 [7, 6]

13 [7, 1]

14 [5, 6]

15 [4]

16 [5, 7]

17 [3, 7]

18 [3, 1]

19 [1, 6]

20 [3, 2]

21 [6, 1]

22 [4, 5, 6]

23 [7, 2]

24 [6, 7, 6]

25 [6, 5]

26 [1, 2]

27 [7, 1, 7]

28 [4, 2]

29 [5, 6, 7]

30 [4, 5]

31 [4, 7]

32 [4, 6]

33 [4, 1]

34 [2, 6]

35 [7, 1, 3]

36 [7, 1, 2]

37 [7, 5]

38 [4, 3]

39 [3, 6]

40 [4, 6, 5]

41 [5, 4, 2]
42 [3, 4, 5]
43 [1, 3]
44 [2, 7, 2]
45 [6, 5, 6]
46 [2, 4]
47 [1, 3, 2]
48 [5, 4]
49 [2, 3]
50 [7, 1, 6]
51 [1, 2, 1]
52 [5, 3]
53 [4, 2, 7]
54 [6, 7, 1]
55 [3, 5, 2]
56 [3, 4]
57 [5, 1]
58 [5, 2]
59 [1, 2, 3]
60 [3, 5]
61 [3, 7, 6]
62 [6, 2]
63 [1, 6, 5]
64 [1, 5]
65 [1, 2, 4]
66 [6, 7, 2]
67 [7, 4]
68 [6, 4]
69 [3, 5, 4]
70 [5, 2, 4]
71 [2, 5]
72 [4, 3, 1]
73 [6, 5, 3]
74 [2, 3, 4]
75 [2, 3, 5]
76 [4, 5, 4]
77 [2, 3, 2]
78 [7, 3]
79 [1, 3, 6]
80 [2, 4, 3]
81 [6, 4, 3]
82 [6, 3]
83 [5, 3, 1]
84 [6, 3, 5]

85 [4, 1, 3]

86 [3, 7, 2]

87 [6, 4, 2]

88 [1, 4]

89 [7, 1, 3, 1]

90 [7, 1, 7, 3]

Define (t_i):

(t_0):

(1, 2)(3, 6)(4, 7)(5, 10)(8, 16)(9, 17)(11, 22)(12, 24)(13, 27)(14, 29)(15, 31)(18, 35)(19, 36)(20, 40)(21, 41)(23, 44)(25, 47)(26, 50)(28, 53)(30, 55)(32, 59)(33, 61)(34, 63)(37, 51)(38, 69)(39, 70)(42, 52)(43, 75)(45, 78)(46, 79)(48, 81)(49, 73)(54, 58)(56, 85)(57, 86)(60, 87)(62, 65)(64, 83)(66, 89)(67, 77)(68, 74)(71, 84)(72, 88)(76, 90)(80, 82)

(t_1):

(1, 3)(2, 13)(4, 21)(5, 11)(6, 27)(7, 54)(8, 57)(9, 18)(10, 59)(12, 80)(14, 63)(15, 33)(16, 74)(17, 66)(19, 77)(20, 29)(22, 68)(23, 73)(24, 88)(25, 55)(26, 51)(28, 65)(30, 70)(31, 84)(32, 86)(34, 87)(35, 89)(36, 82)(37, 42)(38, 72)(39, 53)(40, 48)(41, 71)(43, 76)(44, 64)(45, 90)(46, 81)(47, 56)(49, 50)(52, 83)(58, 61)(60, 79)(62, 85)(67, 69)(75, 78)

(t_2):

(1, 5)(2, 23)(3, 26)(4, 62)(6, 69)(7, 66)(8, 58)(9, 20)(10, 44)(11, 51)(12, 63)(13, 36)(14, 84)(15, 28)(16, 85)(17, 86)(18, 74)(19, 22)(21, 42)(24, 90)(25, 72)(27, 71)(29, 37)(30, 80)(31, 83)(32, 79)(33, 35)(34, 76)(38, 54)(39, 73)(40, 64)(41, 48)(43, 47)(45, 46)(49, 77)(50, 56)(52, 75)(53, 57)(55, 60)(59, 67)(61, 82)(65, 89)(68, 87)(70, 78)(81, 88)

(t_3):

(1, 9)(2, 78)(3, 43)(4, 82)(5, 49)(6, 66)(7, 53)(8, 52)(10, 29)(11, 40)(12, 41)(13, 35)(14, 69)(15, 38)(16, 79)(17, 45)(18, 76)(19, 54)(20, 77)(21, 70)(22, 23)(24, 60)(25, 73)(26, 59)(27, 90)(28, 42)(30, 50)(31, 47)(32, 63)(33, 85)(34, 72)(36, 48)(37, 86)(39, 51)(44, 56)(46, 80)(55, 71)(57, 87)(58, 65)(61, 67)(62, 83)(64, 74)(68, 81)(75, 89)(84, 88)

(t_4):

(1, 15)(2, 67)(3, 88)(4, 68)(5, 46)(6, 73)(7, 40)(8, 48)(9, 56)(10, 36)(11, 35)(12, 47)(13, 83)(14, 50)(16, 66)(17, 41)(18, 54)(19, 85)(20, 72)(21, 79)(22, 52)(23, 84)(24, 33)(25, 59)(26, 65)(27, 32)(28, 45)(29, 43)(30, 76)(31, 77)(34, 42)(37, 55)(38, 44)(39, 63)(49, 74)(51, 90)(53, 87)(27, 58)(28, 36)(29, 68)(31, 66)(32, 40)(33, 73)(34, 61)(38, 41)(39, 83)(42, 56)(43, 53)(44, 57)(46, 54)(48, 76)(49, 75)(55, 89)(62, 69)(67, 81)(77, 90)(78, 85)(82, 84)(86, 88)

(t_6):

(1, 4)(2, 12)(3, 19)(5, 34)(6, 42)(7, 24)(8, 14)(9, 39)(10, 66)(11, 69)(13, 50)(15, 32)(16, 72)(17, 61)(18, 84)(20, 80)(21, 77)(22, 30)(23, 65)(25, 45)(26, 41)(27, 68)(28, 74)(29, 33)(31, 87)(35, 57)(36, 60)(37, 54)(38, 75)(40, 78)(43, 79)(44, 90)(46, 83)(47, 58)(48, 73)(49, 86)(51, 82)(52, 59)(53, 67)(55, 64)(56, 81)(62, 76)(63, 89)(70, 88)(71, 85)

The group is defined by the symmetric presentation. It index is at most:

$$\frac{|N|}{|N|}+\frac{|N|}{|N^{(0)}|}+\frac{|N|}{|N^{(01)}|}+\frac{|N|}{|N^{(010)}|}+\frac{|N|}{|N^{(012)}|}+\frac{|N|}{|N^{(013)}|}+\frac{|N|}{|N^{(0103)}|}+\frac{|N|}{|N^{(0131)}|}$$

$= 1 + 7 + 42 + 7 + 24 + 7 + 1 + 1 = 90$

Since the map is completed, at this point G has order at most

$|L_3(2)|(1+7+42+7+24+7+1+1) = 168 \times 90 = 15120$

Since the group $3^{\cdot}S_7$ is generated by x, y, and t

$\Rightarrow$ $3^{\cdot}S_7$ is an image of G.

$\Rightarrow$ $|G| \geq |3^{\cdot}S_7|$

$\Rightarrow |G| \geq 15120$

$\Rightarrow 15120 \leq |G| \leq 15120$

$\Rightarrow |G| = 15120$

$\Rightarrow G \cong 3^{\cdot}S_7$

CHAPTER FIVE

SYMMETRIC GENERATION OF $2^{3_1}\, 2^{3_1+3_2} : L_3(2)$

Symmetric Presentation

Symmetric presentations for 2^{*7}: $L_3(2)$ are given by

$<x,y,t \mid x^7 = y^2 = (xy)^3 = [x,y]^4 = 1 = t^2 = [xy, t^{x^4}] = [y, t^{x^3}]>$,

where $L_3(2) = <x,y>$, $x \sim (0,1,2,3,4,5,6)$, $y \sim (2,6)(4,5)$. We factor by the relation $[(0\ 1\ 2\ 3\ 4\ 5\ 6)\ t_0]^7$ to obtain

$$G = \frac{2^{*7} : L_3(2)}{[(0123456)t_0]^7} \cong 2^{3_1}\, 2^{3_1+3_2} : L_3(2) \text{ (see [4]).}$$

Let $N = L_3(2)$. Now N is transitive on the set of symmetric generators $\omega = \{0,1,..,6\}$. The double coset Nt_iN contains 7 single cosets, since

$$N^0 = \langle (1,2,3,4,5,6,7),\ (2,6)(4,5) \rangle$$

is transitive on {1,2,3,4,5,6}, and its orbits on ω are {0}, {1,2,3,4,5,6}.

The relation $[\pi t_0]^7 = 1$, where $\pi = (0\ 1\ 2\ 3\ 4\ 5\ 6)$ gives

$\pi\, t_0\, \pi\, t_0\, \pi\, t_0\, \pi\, t_0\, \pi\, t_0\, \pi\, t_0\, \pi\, t_0 = \pi^7\, \pi^{-6}\, t_0\, \pi^6\, \pi^{-5}\, t_0\, \pi^5\, \pi^{-4}\, t_0\, \pi^4\, \pi^{-3}\, t_0\, \pi^3\, \pi^{-2}\, t_0\, \pi^2\, \pi^{-1}\, t_0\, \pi\, t_0$

$\Rightarrow [(0\ 1\ 2\ 3\ 4\ 5\ 6)\ t_0]^7 = [\pi\, t_0]^7 = t_6\, t_5\, t_4\, t_3\, t_2\, t_1\, t_0$

We also use other relations,

$[(1\ 2\ 4)(3\ 5\ 6)\ t_1]^{12} = t_4\, t_2\, t_1\, t_4\, t_2\, t_1\, t_4\, t_2\, t_1\, t_4\, t_2\, t_1$

$[(1\ 2\ 4)(3\ 5\ 6)\ t_3]^{12} = t_5\ t_6\ t_3\ t_5\ t_6\ t_3\ t_5\ t_6\ t_3\ t_5\ t_6\ t_3$

$[(2\ 6)(4\ 5)\ t_2]^8 = t_6\ t_2\ t_6\ t_2\ t_6\ t_2\ t_6\ t_2$

$[(1\ 3)(2\ 4\ 6\ 5)\ t_1]^8 = t_3\ t_1\ t_3\ t_1\ t_3\ t_1\ t_3\ t_1$

$[(1\ 3)(2\ 4\ 6\ 5)\ t_2]^{16} = t_5\ t_6\ t_4\ t_2\ t_5\ t_6\ t_4\ t_2\ t_5\ t_6\ t_4\ t_2\ t_5\ t_6$
$t_4\ t_2$

$[(2\ 6)(4\ 5)\ t_0\ t_2]^8 = t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2\ t_0\ t_6\ t_0\ t_2\ t_0\ t_6$
$t_0\ t_2$

$[(2\ 6)(4\ 5)\ t_1\ t_2]^8 = t_1\ t_6\ t_1\ t_2\ t_1\ t_6\ t_1\ t_2\ t_1\ t_6\ t_1\ t_2\ t_1\ t_6$
$t_1\ t_2$

Since $[\pi\ t_0]^7 = t_6\ t_5\ t_4\ t_3\ t_2\ t_1\ t_0$ = Identity (e)

$\Rightarrow (7\ 1\ 2\ 3\ 4\ 5\ 6) = e$

(letting $t_6, t_5, t_4, t_3, t_2, t_1$, and t_0 be 6,5,4,3,2,1,7)

Conjugate (7 1 2 3 4 5 6) with n in N. We have,

$\underline{5}\ 3\ 4\ 7\ 1\ \underline{2\ 6} = e = 4\ 7\ 3\ 5\ 1\ 2\ 6 = \underline{5}\ 7\ 1\ 4\ 3\ \underline{2\ 6} = e$

$= 1753426 = e.$

$\underline{3\ 4}\ 7\ 1 = 7\ 1\ 4\ \underline{3};\ 7\ 1\ 3 = \underline{4}\ 3\ 7\ 1\ \underline{4};\ 3\ 7\ 1 = 4\ 7\ 1\ 3\ 4$

Conjugate to n in N, we have:

$3\ 7\ 1 = \underline{4}\ 7\ 1\ 3\ \underline{4}$ and $3\ 7\ 1 = \underline{4}\ 1\ 3\ 7\ \underline{4}$

$\Rightarrow 3\ 7\ 1 = 7\ 1\ 3 = 1\ 3\ 7.$

Thus,

$3\ 7\ 1 \sim 7\ 1\ 3 \sim 1\ 3\ 7$

Since $7\ 1\ \underline{3} = 1\ 3\ 7 \Rightarrow 7\ 1\ 7 = \underline{1\ 3\ 7}\ 3\ 7$

$\Rightarrow 7\ 1\ 7 = 3\ 7\ \underline{1\ 3\ 7}$ (Since $1\ 3\ 7 = 3\ 7\ 1$)

$7\ 1\ 7 = 3\ \underline{7\ 7}\ 1\ 3 \Rightarrow 7\ 1\ 7 = 3\ 1\ 3$

Thus,

$7\ 1\ 7 \sim 3\ 1\ 3$

Since $7\ 1\ \underline{3} = 1\ 3\ 7 \Rightarrow 7\ 1 = 1\ 3\ 7\ \underline{3}$

$\Rightarrow 7\ 1\ 7\ 1 = \underline{1\ 3\ 7}\ 3\ 7\ 1 = 3\ 7\ 1\ \underline{3\ 7\ 1} = 3\ 7\ 1\ 1\ 3\ 7$

(Since $1\ 3\ 7 = 3\ 7\ 1 = 7\ 1\ 3$) $\Rightarrow 7\ 1\ 7\ 1 = 3\ 7\ 3\ 7$

And $7\ 1\ 3 = 1\ 3\ 7 \Rightarrow 7\ 1 = 1\ 3\ 7\ 3$

$\Rightarrow 7\ 1\ 7\ 1 = \underline{1\ 3\ 7\ 3\ 7\ 1} = 7\ \underline{1\ 3\ 7}\ 1\ 3$

(since $7\ 1\ 3 = 3\ 7\ 1 = 1\ 3\ 7$)

$\Rightarrow 7\ 1\ 7\ 1 = 7\ 3\ 7\ \underline{1\ 1}\ 3$ (same as above)

$\Rightarrow 7\ 1\ 7\ 1 = 7\ 3\ 7\ 3$

Thus,

$7\ 1\ 7\ 1 \sim 3\ 7\ 3\ 7 \sim 7\ 3\ 7\ 3$

Since $7\ 1\ 7 = 3\ 1\ 3 \Rightarrow 1\ 2\ 1 = 4\ 2\ 4$

(conjugate with n in N)

$\Rightarrow 7\ 1\ 2\ 1 = 7\ 4\ 2\ 4$

Thus, $7\ 1\ 2\ 1 \sim 7\ 4\ 2\ 4$

Since $7\ 1\ 3 = 1\ 3\ 7 = 3\ 7\ 1$ Conjugate to n in N, we have

$4\ 1\ 2 = 1\ 2\ 4 = 2\ 4\ 1 \Rightarrow 7\ 4\ 1\ 2 = 7\ 1\ 2\ 4 = 7\ 2\ 4\ 1$

Thus,

$7\ 4\ 1\ 2 \sim 7\ 1\ 2\ 4 \sim 7\ 2\ 4\ 1$

Since $7\ 1\ 3 = 3\ 7\ 1 = 1\ 3\ 7$ and $7\ 1\ 3 = 5\ 1\ 3\ 7\ 5$

$\Rightarrow 7\ 1\ \underline{3} = \underline{5}\ 7\ 1\ 3\ 5$ (replace $1\ 3\ 7$ by $7\ 1\ 3$)

$\Rightarrow 5\ 7\ 1 = 7\ 1\ 3\ 5\ 3 \Rightarrow 5\ 7\ 1\ 2 = \underline{7\ 1\ 3}\ 5\ 3\ 2$

$\Rightarrow 5\ 7\ 1\ 2 = 2\ \underline{7\ 1\ 3}\ 2\ 5\ 3\ 2 = 2\ 5\ 7\ 1\ \underline{3\ 5\ 2\ 5\ 3\ 2}$

$\Rightarrow 5\ 7\ 1\ 2 = 2\ 5\ 7\ 1$ (since 3 5 2 5 3 2 equals identity.)

Similarly, $7\ 1 = 1\ 3\ 7\ 3 \Rightarrow 7\ 1\ 2\ 5 = \underline{1\ 3\ 7}\ 3\ 2\ 5$

$\Rightarrow 7\ 1\ 2\ 5 = 2\ \underline{1\ 3\ 7}\ 2\ 3\ 2\ 5$

$\Rightarrow 7\ 1\ 2\ 5 = 2\ 5\ \underline{1\ 3\ 7}\ 5\ 2\ 3\ 2\ 5 = 2\ 5\ 7\ 1\ \underline{3\ 5\ 2\ 3\ 2\ 5}$

$\Rightarrow 7\ 1\ 2\ 5 = 2\ 5\ 7\ 1$ since 3 5 2 3 2 5 equals identity.

Conjugate to n in N, $(5\ 7\ 1\ 2)^n = (2\ 5\ 7\ 1)^n = (7\ 1\ 2\ 5)^n$

We have: $2\ 5\ 7\ 1 = 1\ 2\ 5\ 7 = 5\ 7\ 1\ 2$

Thus,

$7\ 1\ 2\ 5 \sim 1\ 2\ 5\ 7 \sim 2\ 5\ 7\ 1 \sim 5\ 7\ 1\ 2$

Since $1\ 2\ 5\ \underline{7} \sim \underline{5}\ 7\ 1\ 2 \Rightarrow 5\ 1\ 2\ 5 \sim 7\ 1\ 2\ 7$

Conjugate by n in N, $(5\ 1\ 2\ 5)^n \sim (7\ 1\ 2\ 7)^n$, we have

$6\ 2\ 1\ 6 \sim 3\ 2\ 1\ 3$

Since $\underline{3}\ 7\ 1 = 2\ 1\ 3\ \underline{7\ 2}$ ($e = 1\ 2\ 3\ 7\ 6\ 4\ 5 = 7\ 2\ 1\ 3\ 4\ 2\ 5$)

$\Rightarrow 7\ 1\ 2\ 7 = 3\ 2\ 1\ 3$

Thus,

$6\ 2\ 1\ 6 \sim 3\ 2\ 1\ 3 \sim 5\ 1\ 2\ 5 \sim 7\ 1\ 2\ 7$

Similarly, $1\ 7\ 2\ 1\ 7 \sim 7\ 1\ 2\ 7\ 1$ since $6\ 2\ 6\ 2\ 6\ 2\ 6\ 2 = e$,

conjugate with n in N

$\Rightarrow 1\ 7\ 1\ 7\ 1\ 7\ 1\ 7 = e = 7\ 1\ 7\ 1\ 7\ 1\ 7\ 1$

Since $5\ 1\ 2\ 7 = 2\ 7\ 5\ 1 \Rightarrow \underline{5\ 1}\ 2\ 7\ 1\ 5\ 7\ 2 = e$

$\Rightarrow \underline{5\ 7}\ 7\ 1\ 2\ 7\ 1\ 5\ 7\ \underline{2} = e \Rightarrow 7\ 5\ 2 = 7\ 1\ 2\ 7\ 1\ \underline{5\ 7}$

$\Rightarrow$ 7 5 2 7 5 = 7 1 2 7 1

Thus,

7 5 2 7 5 ~ 7 1 2 7 1 ~ 1 7 2 1 7

Similarly, 3 5 7 5 2 ~ 2 6 3 6 7 ~ 7 1 2 1 3

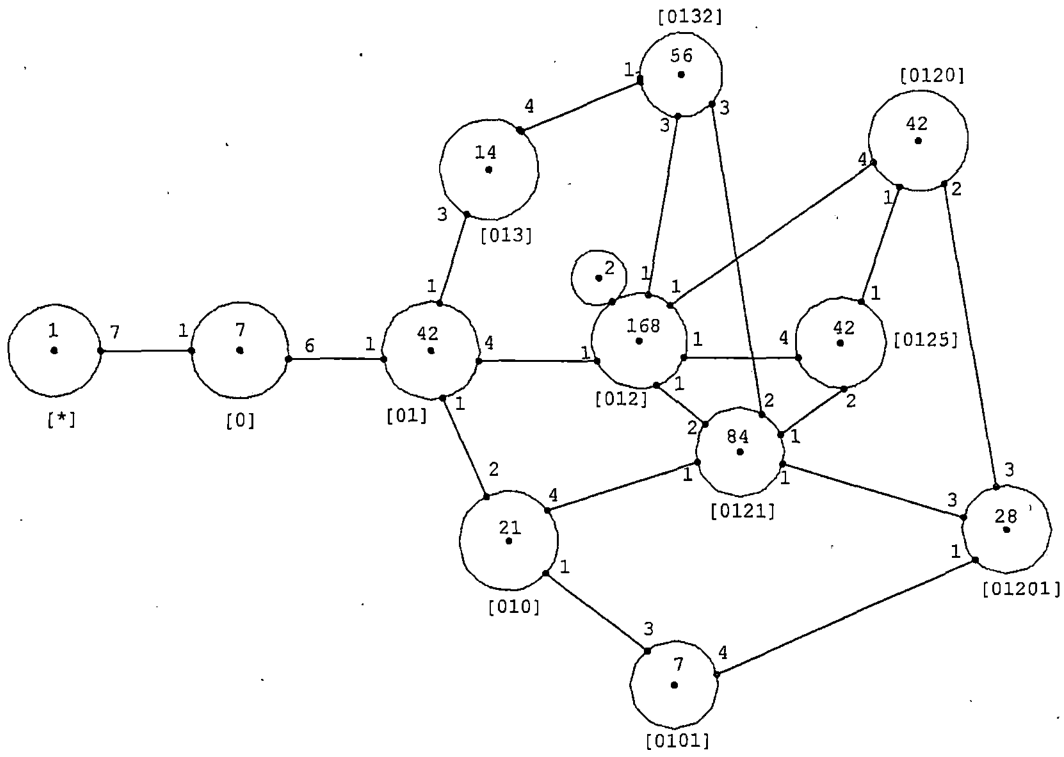

Figure 5.1. Cayley Graph of $2^{3_1} 2^{3_1+3_2} : L_3(2)$ over $L_3(2)$

The Identification of Cosets Labeling

MAGMA Labeling - Single coset

1 [*]	4 [6]
2 [7]	5 [2]
3 [1]	6 [1, 7]

7 [6, 7]
8 [5]
9 [3]
10 [2, 7]
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44 [1, 3]
45 [7, 2, 7]
46 [3, 7, 3]
47 [1, 5, 1]
48 [2, 4]
49 [6, 5, 7]
50 [5, 4]
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504 [2, 4, 3, 2, 4]

505 [6, 7, 3, 6]

506 [5, 2, 4, 6]

507 [1, 2, 7, 1, 2]

508 [7, 1, 4, 7]

509 [1, 2, 3, 1, 2]

510 [4, 7, 3, 4]

511 [1, 2, 6, 1]

512 [3, 5, 7, 5]

The group is defined by the symmetric presentation. Its index is at most:

$$\frac{|N|}{|N|}+\frac{|N|}{|N^{(0)}|}+\frac{|N|}{|N^{(01)}|}+\frac{|N|}{|N^{(010)}|}+\frac{|N|}{|N^{(012)}|}+\frac{|N|}{|N^{(013)}|}+\frac{|N|}{|N^{(0101)}|}+\frac{|N|}{|N^{(0120)}|}+\frac{|N|}{|N^{(0121)}|}+\frac{|N|}{|N^{(0125)}|}+\frac{|N|}{|N^{(0132)}|}+\frac{|N|}{|N^{(01201)}|}$$

= 1+ 7 + 42 + 21 + 168 + 14 + 7 + 42 + 84 + 42 + 56 + 28

= 512

Since the map is completed, at this point G has order at most,

$|L_3(2)|$(1+7+42+21+7+168+14+56+42+84+42+28)

= 168 x 512 = 86016

Since the group $2^{3_1}\, 2^{3_1+3_2}: L_3(2)$ is generated by x, y, and t

$\Rightarrow$ $2^{3_1}\, 2^{3_1+3_2}: L_3(2)$ is an image of G.

$\Rightarrow$ $|G| \geq |2^{3_1}\, 2^{3_1+3_2}: L_3(2)|$ $\Rightarrow$ $|G| \geq 86016$

$\Rightarrow$ $86016 \leq |G| \leq 86016 \Rightarrow |G| = 86016$

$\Rightarrow$ $G \cong 2^{3_1}\, 2^{3_1+3_2}: L_3(2)$

CHAPTER SIX

SYMMETRIC GENERATION OF J_3: 2

Symmetric Presentation

Symmetric presentations of 2^{*120}: $L_2(16)$: 4 are given by

$L_2(16)$: 4 <x,y,z> := Group < x,y,z| $x^{17} = y^8 = (x^y)*x^{-2} = z^2 = y^z*y^{-5} = (x*y*z)^4 = (x*z)^{17} = 1$>

The progenitor factored by the relation $(x^3*y^5*z*t)^5 = 1$

$$G = \frac{2^{*120}: L_2(16):4}{[(x^3y^5z)t]^5} \cong J_3: 2 \text{ (see [2])}$$

J_3: 2 can be symmetrically represented by <x,y,z,t; x^17 = y^8 = 1, x^y = x^2, z^2 = 1, y^z = y^5, (x*y*z)^4 = 1, (x*z)^17 = 1 = t^2 = [x,t] = [y,t] =(x^3*y^5*z*t)^5> = J_3: 2

Coset Action: Let N = $L_2(16)$: 4. First of all, we need to compute $L_2(16)$: 4 to obtain permutations of x, y, and z such as: $L_2(16)$: 4 = <x,y,z> is a permutation group on the cosets of the subgroup <x,y>.

S120 ~ Sym(120)

x ~ (2, 3, 5, 11, 12, 27, 25, 24, 10, 7, 18, 38, 41, 20, 21, 8, 4)(6, 14, 28, 54, 64, 100, 56, 55, 29, 23, 9, 22, 43, 78, 72, 36, 16)(13, 19, 40, 74, 37, 32, 62, 108, 84, 47, 45, 66, 34, 17, 15, 33, 30)(26, 50, 88, 97, 73, 99, 119, 114, 118, 120, 102, 70, 98, 113, 77, 42, 52)(31, 58, 96, 53, 68, 111, 110, 94, 117, 109, 65, 76, 115, 83, 46, 82, 60)(35, 67, 81, 44, 80, 63, 107, 87, 49, 86, 93, 92, 51, 91, 89, 61, 69)(39, 48, 85, 71, 112, 103, 57, 101, 116, 95, 90, 106, 59, 105, 104, 79, 75)

$y \sim (3, 5, 12, 10, 4, 8, 20, 7)(6, 15, 28, 19, 9, 13, 29, 17)(11, 25, 41, 24, 21, 38, 27, 18)(14, 30, 56, 47, 23, 33, 64, 32)(16, 34, 36, 45, 22, 40, 43, 37)(26, 51, 76, 39, 42, 49, 53, 48)(31, 59, 99, 63, 46, 57, 102, 61)(35, 68, 71, 73, 44, 65, 79, 70)(50, 89, 82, 116, 77, 107, 60, 90)(52, 93, 94, 95)(54, 74, 72, 84, 55, 66, 78, 62)(58, 104, 120, 91, 83, 112, 119, 87)(67, 110, 101, 98, 81, 117, 106, 97)(69, 96, 75, 113, 80, 115, 85, 88)(86, 111, 103, 118, 92, 109, 105, 114)(100, 108)$

$z \sim (1, 2)(3, 6)(4, 9)(5, 13)(7, 19)(8, 15)(10, 17)(11, 26)(12, 28)(14, 31)(16, 35)(18, 39)(20, 29)(21, 42)(22, 44)(23, 46)(24, 48)(25, 49)(27, 53)(30, 57)(32, 63)(33, 59)(34, 65)(36, 71)(37, 73)(38, 51)(40, 68)(41, 76)(43, 79)(45, 70)(47, 61)(50, 77)(52, 94)(54, 97)(55, 98)(56, 99)(60, 82)(62, 101)(64, 102)(66, 67)(69, 109)(72, 110)(74, 81)(75, 114)(78, 117)(80, 111)(84, 106)(85, 118)(86, 88)(87, 91)(92, 113)(93, 95)(96, 103)(100, 108)(104, 112)(105, 115)$

$N = L_2(16):4$ can be generated by x, y, and z,

where $|x| = 17$, $|y| = 8$, and $|z| = 2$, and

$(x^y)x^{-2} = z^2 = y^z y^{-5} = (xyz)^4 = (xz)^{17} = 1$

Double coset Enumeration:

We factor by $(x^3*y^5*z*t)^5$ to obtain the relations.

The relation gives $(x^3y^5z)^5\, t_{70}\, t_{23}\, t_{51}\, t_2\, t_1 = Id = e$,

where t represents t_1 .

N is transitive on the symmetric generators,

$\omega = \{1,2,..,120\} \Rightarrow Nt_iN = \{Nt_i^{\,n} | n \in N\} = \{ Nt_1,.., Nt_{120}\}$

$N^{(1)}$ = Stabiliser (N,1). $\Rightarrow$ the number of single cosets in

Nt_iN is 120.

$N^{(1)} = \langle a, b \rangle$ is transitive on $\{2,3,4,..,120\}$

The orbits of $N^{(1)}$ on ω are

{ 1 },
{ 2, 3, 4, 5, 7, 8, 10, 11, 12, 18, 20, 21, 24, 25, 27, 38, 41 },
{ 6, 9, 13, 14, 15, 16, 17, 19, 22, 23, 28, 29, 30, 32, 33, 34, 36, 37, 40, 43, 45, 47, 54, 55, 56, 62,64, 66, 72, 74, 78, 84, 100, 108 },
{ 26, 31, 35, 39, 42, 44, 46, 48, 49, 50, 51, 52, 53, 57, 58, 59, 60, 61, 63, 65, 67, 68, 69, 70, 71, 73, 75, 76, 77, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102,103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120 }

We need to see if Nt_1t_2N, Nt_1t_6N, and $Nt_1t_{26}N$ are distinct double cosets and compute number of single cosets in each of these double cosets.

Let d be $(x^3y^5z) \Rightarrow d^5 = (x^3y^5z)^5$

we obtain the relation: $d^5\ t_{70}\ t_{23}\ t_{51}\ t_2\ t_1 = Id$ (e is the identity).

Since for all n in N, $(d^5\ t_{70}\ t_{23}\ t_{51}\ t_2\ t_1)^n = Id$

We have, $d^5\ t_{70}\ t_{23}\ t_{51}\ t_2\ t_1 = (d^5\ t_{70}\ t_{23}\ t_{51}\ t_2\ t_1)^{p8} = Id$

$(d^5)^{p8}\ t_1\ t_2\ t_{51}\ t_{45}\ t_{46} = Id = d^5\ t_{70}\ t_{23}\ t_{51}\ t_2\ t_1$

Substitute $d^5\ t_{70}\ t_{23} = t_1\ t_2\ t_{51}$ on left side,

$(d^5)^{p8}\ d_5\ t_{70}\ t_{23}\ t_{45}\ t_{46} = Id$,

Using d^5 70 23 51 2 1 instead $d^5\ t_{70}\ t_{23}\ t_{51}\ t_2\ t_1$ (the relation given, d^5 70 23 51 2 1 = Identity).

$\Rightarrow (d^5\ 70\ 23\ 51\ 2\ 1)^n = (d^5)^{p8}\ 1\ 2\ 51\ 45\ 46$ (for n = p8 in N)

Since d^5 70 23 = 1 2 51 and 1 2 51 = $((d^5)^{p8})^{(-1)}$ 46 45

$\Rightarrow d^5\ 70\ 23 = ((d^5)^{p8})^{(-1)}\ 46\ 45 \Rightarrow ((d^5)^{p8})d5\ 70\ 23 = 46\ 45$

Let $p_1 = (d^5)^{p8}d^5 \Rightarrow (((d^5)^{p8})d^5\ 70\ 23)^n = (46\ 45)^n$

$\Rightarrow (p_1)^{p45}\ 1\ 2 = 100\ 108$

and $(p_1)^{p46}\ 1\ 2 = 108\ 100$ (for p_{45} and p_{46} in N).

$\Rightarrow (p_1{}^{p45})^{(-1)}\ 100\ 108 = 1\ 2 = (p_1{}^{p46})^{(-1)}\ 108\ 100$

$\Rightarrow (p_1{}^{p45})(p_1{}^{p46})^{(-1)}\ 108\ 100 = 100\ 108$

Let $p_2 = (p_1{}^{p45})(p_1{}^{p46})^{(-1)}$

$\Rightarrow (p_2\ 108\ 100)^n = (100\ 108)^n$

$\Rightarrow p_3{}^{p59}\ 2,1 = 1,2$ (for p59 in N)

$\Rightarrow 1\ 2 = p_2{}^{p59}\ 2\ 1 = (p_1{}^{p46})^{(-1)}\ 108\ 100 = (p_1{}^{p45})^{(-1)}\ 100\ 108$

Thus,

$1\ 2 \sim 2\ 1 \sim 100\ 108 \sim 108\ 100$

Similarly,

$p_1\ 70\ 23\ 45\ 46 = e \Rightarrow (p_1\ 70\ 23\ 45\ 46)^n = e$

$\Rightarrow (p_1)^{q22}\ 40\ 6\ 1\ 41 = e$ (for q_{22} in N)

Let $q_1 = p_1{}^{q22} \Rightarrow 41\ q_1\ 40 = 1\ 6 \Rightarrow q_1\ 1\ 40 = 1\ 6$

Thus,

$1\ 6 \sim 1\ 40$

Similarly,

Using the relation $1\ 2 = (p_1{}^{p46})^{-1}\ 108\ 100$. Let $a_3 = (p_1{}^{p46})^{-1}$

$\Rightarrow 1\ 2 = a_3\ \underline{108}\ 100 \Rightarrow 1\ 2 = \underline{100}\ a_3\ 100$

$\Rightarrow 100\ 1\ 2 = a_3\ 100 \Rightarrow \underline{41}\ 100\ 1\ 2 = \underline{41}\ a_3\ 100$

$\Rightarrow 41\ 100\ 1\ 2 = a_3\ \underline{26\ 100}$ (1) $\Rightarrow (a_3)^{-1}\ \underline{41\ 100}\ 1\ 2\ 100\ 26 = e$

$\Rightarrow (a_3)^{-1} a_1{}^{j}\ \underline{100}\ 41\ 1\ 2\ \underline{100}\ 26 = e$ (Since $(1\ 2)^{j} = 41\ 100$ and $1\ 2 = a_1\ 2\ 1$.)

Let $j_1 = j_3{}^{-1} (a_3)^{-1} a_1{}^{j} \Rightarrow j_1\ 2\ 41\ 100\ 1\ 2\ 107 = e$ (2)

$\Rightarrow (j_1)^{-1} = (a_1{}^{j})^{-1} a_3\ (j_3) \Rightarrow (j_1)^{-1} = 2\ 41\ 100\ 1\ 2\ 107$

Since $41\ 100\ 1\ 2 = a_3\ 26\ 100$ (by (1)) $\Rightarrow \underline{2}\ 41\ 100\ 1\ 2\ \underline{107} = \underline{2}\ a_3\ 26\ 100\ \underline{107} \Rightarrow 2\ 41\ 100\ 1\ 2\ 107 = a_3\ 1\ 26\ 100\ 107$

$\Rightarrow (j_1)^{-1} = a_3\ 1\ 26\ 100\ 107$ (by (2))

$\Rightarrow e = (j_1)a_3\ 1\ 26\ 100\ 107$ and $e = ((j_1)a_3)^{j4}\ 1\ 26\ 55\ 69$

(letting $r_1 = (((j_1)a_3)^{j4})^{-1}$ and $r_2 = ((j_1)a_3)^{-1}$

$\Rightarrow 1\ 26 = r_1\ 69\ 55 = r_2\ 107\ 100$.)

Thus,

$1\ 26 \sim 69\ 55 \sim 107\ 100$

Since $d^5\ 70\ 23\ 51\ 2\ 1 = e \Rightarrow 1\ 2\ 51 = d^5\ 70\ 23$

Since $(70\ 23)^n = (1\ 2) \Rightarrow 1\ 2\ 51$ is in orbit $1\ 2$

Since 51 is in the set

$\{11,18,21,24,25,26,27,38,39,41,42,48,49,51,53,76\}$

$\Rightarrow$ 1 2 11, 1 2 18, 1 2 21, 1 2 24, 1 2 25, 1 2 26, 1 2 27, 1 2 38, 1 2 39, 1 2 41, 1 2 42, 1 2 48, 1 2 49, 1 2 51, 1 2 53, and 1 2 76 are all in orbit 1 2.

Since $d^5\ 70\ 23\ 51\ 2\ 1 = e \Rightarrow d^5\ 70\ 23\ (p_2{}^{p59})^{q01}\ 2\ 51\ 1 = e$

$\Rightarrow d^5 (p_2{}^{p59})^{q01}\ 61\ 80\ 2\ 51\ 1 = e$

$\Rightarrow d^5 (p_2{}^{p59})^{q01} (p_2{}^{p59})^{q02}\ 80\ 61\ 2\ 51\ 1$

$\Rightarrow (d^5 (p_2{}^{p59})^{q01} (p_2{}^{p59})^{q02}\ 80\ 61\ 2\ 51\ 1)^{q03} = e$

$\Rightarrow$ $(d^5\ (p_2{}^{p59})^{q01}(p_2{}^{p59})^{q02})^{q03}$ 27 1 6 97 68 = e

Let $r = ([d^5(p_2{}^{p59})^{q01}(p_2{}^{p59})^{q02}]^{q03})^{-1}$

$\Rightarrow$ 1 6 = 21 r 68 97 $\Rightarrow$ 1 6 = r 80 68 97

Since $(80\ 68)^n$ is in orbit 1 26 $\Rightarrow$ 1 6 97 is in 1 26

Since 97 in the set {31,39,68,80,88,97,107,116}

$\Rightarrow$ 1 6 31, 1 6 39, 1 6 68, 1 6 80, 1 6 88, 1 6 97, 1 6 107, and 1 6 116 are all in the orbit 1 26

Since d^5 70 23 51 2 1 = e $\Rightarrow$ $(d^5\ 70\ 23\ 51\ 2\ 1)^{r3}$ = e

$\Rightarrow$ $d^{5\ r3}$ 1 2 26 36 63 = e $\Rightarrow$ $d^{5\ r3}(dp2^{p59})$ 2 1 26 36 63 =

$\Rightarrow$ 64 $d^{5\ r3}(p_2{}^{p59})$ 1 26 36 63 = e

$\Rightarrow$ $d^{5\ r3}(p_2{}^{p59})$ 1 26 = 64 63 36

Let $r_1 = [d^{5\ r3}(p_2{}^{p59})]^{-1}$ then 1 26 36 = r1 64 63

Since $(64\ 63)^n$ is in orbit 1 26 $\Rightarrow$ 1 26 36 is in 1 26

Since 36 is in the set {36,72,76,102,111,114} $\Rightarrow$ 1 26 36, 1 26 72, 1 26 76, 1 26 102, 1 26 111, and 1 26 114 are all in the orbit 1 26

Since d_5 70 23 51 2 1 = e $\Rightarrow$ d_5 70 $(p_2{}^{p59})^{r23}$ 51 23 2 1 = e

$\Rightarrow$ $d^5\ (p_2{}^{p59})^{r23}$ 83 51 23 2 1 = e

$\Rightarrow$ $d^5\ (p_2{}^{p59})^{r23}$ 83 51 = 1 2 23 Since $(83\ 51)^n$ is in orbit 1 26 $\Rightarrow$ 1 2 23 is in 1 26

Since 23 is in the set

{14,16,22,23,30,31,32,33,34,35,36,37,40,43,44,45,46,47,56, 57,59,61,63,64,65,68,70,71,73,79,99,102}

$\Rightarrow$ 1 2 14, 1 2 16, 1 2 22, 1 2 23, 1 2 30, 1 2 31, 1 2 32, 1 2 33, 1 2 34, 1 2 35, 1 2 36, 1 2 37, 1 2 40, 1 2 43, 1 2 44, 1 2 45, 1 2 46, 1 2 47, 1 2 56, 1 2 57, 1 2 59, 1 2 61, 1 2 63, 1 2 64, 1 2 65, 1 2 68, 1 2 70, 1 2 71, 1 2 73, 1 2 79, 1 2 99, and 1 2 102 are all in orbit 1 26

Since d^5 70 23 51 2 1 = e $\Rightarrow$ $d_5{}^{r13}$ 79 47 26 2 1 = e

$\Rightarrow$ $d_5{}^{r13}$ 79 = 1 2 26 47 = $p_2{}^{p59}$ 2 1 26 47

$\Rightarrow$ 2 $(p_2{}^{p59})^{-1}$ $d5^{r13}$ 79 = 1 26 47 = $p_2{}^{p59}d5^{r13}$ 16 79

Since $(16\ 79)^n$ is in orbit 1 26 $\Rightarrow$ 1 26 47 is in orbit 1 26

Since 47 is in the set {21, 27, 47, 73, 108, 113}

$\Rightarrow$ 1 26 21, 1 26 27, 1 26 47, 1 26 73, 1 26 108, and 1 26 113 are all in the orbit 1 26.

Similarly,

N 1 6 41 63 112 = e conjugate with n in N

$\Rightarrow$ N 94 93 52 2 1 = e

N 1 26 107 5 93 = e conjugate with n in N

$\Rightarrow$ N 48 13 50 2 1 = e

N 1 6 103 98 52 = e conjugate with n in N

$\Rightarrow$ N 104 62 3 2 1 = e

Thus,

1 2 52 ~ 94 93 $\Rightarrow$ 1 2 52 $\in$ orbit 1 6

1 2 50 ~ 48 13 $\Rightarrow$ 1 2 50 $\in$ orbit 1 26

1 2 3 ~ 104 62 $\Rightarrow$ 1 2 3 $\in$ orbit 1 6

Since 1 6 31 ~ 112 44 $\Rightarrow$ 1 6 31 $\in$ orbit 1 26

and 1 6 41 ~ 63 112 (conjugate with n in N from (1 2 52 ~ 94 93)^n $\Rightarrow$ 63 112 41 ~ 1 6) $\Rightarrow$ 1 6 41 $\in$ orbit 1 2

We obtain: N 1 26 85 43 69 = e conjugate with n in N

$\Rightarrow$ N 99 66 63 6 1 = e

N 1 6 51 120 84 = e conjugate with n in N

$\Rightarrow$ N 30 89 2 6 1 = e

N 1 26 39 99 32 = e conjugate with n in N

$\Rightarrow$ N 108 15 4 6 1 = e

N 1 26 104 33 72 = e conjugate with n in N

$\Rightarrow$ N 13 10 9 6 1 = e

N 1 6 73 36 47 = e conjugate with n in N

$\Rightarrow$ N 45 67 13 6 1 = e

N 1 26 22 31 32 = e conjugate with n in N

$\Rightarrow$ N 43 36 17 6 1 = e

N 1 26 40 36 97 = e conjugate with n in N

$\Rightarrow$ N 39 7 19 6 1 = e

N 1 26 83 59 95 = e conjugate with n in N

$\Rightarrow$ N 110 70 26 6 1 = e

N 1 26 20 113 87 = e conjugate with n in N

$\Rightarrow$ N 112 4 31 6 1 = e

N 1 26 7 114 119 = e conjugate with n in N

$\Rightarrow$ N 89 17 44 6 1 = e

N 1 6 78 61 16 = e conjugate with n in N

$\Rightarrow$ N 72 96 49 6 1 = e

N 1 6 105 29 66 = e conjugate with n in N

$\Rightarrow$ N 55 84 51 6 1 = e

N 1 2 6 25 112 = e conjugate with n in N

$\Rightarrow$ N 59 90 65 6 1 = e

Thus,

1 6 63 ~ 99 66 $\Rightarrow$ 1 6 63 $\in$ orbit 1 26

1 6 2 ~ 30 89 $\Rightarrow$ 1 6 2 $\in$ orbit 1 6

1 6 4 ~ 108 15 $\Rightarrow$ 1 6 4 $\in$ orbit 1 26

1 6 9 ~ 13 10 $\Rightarrow$ 1 6 9 $\in$ orbit 1 26

1 6 13 ~ 45 67 $\Rightarrow$ 1 6 13 $\in$ orbit 1 6

1 6 17 ~ 43 36 $\Rightarrow$ 1 6 17 $\in$ orbit 1 26

1 6 19 ~ 39 7 $\Rightarrow$ 1 6 19 $\in$ orbit 1 26

1 6 26 ~ 110 70 $\Rightarrow$ 1 6 26 $\in$ orbit 1 26

1 6 44 ~ 89 17 $\Rightarrow$ 1 6 44 $\in$ orbit 1 26

1 6 49 ~ 72 96 $\Rightarrow$ 1 6 49 $\in$ orbit 1 6

1 6 51 ~ 55 84 $\Rightarrow$ 1 6 51 $\in$ orbit 1 6

1 6 65 ~ 59 90 $\Rightarrow$ 1 6 65 $\in$ orbit 1 2

Since 1 26 17 ~ 15 64 $\Rightarrow$ 1 26 17 $\in$ orbit 1 6

1 26 36 ~ 64 63 $\Rightarrow$ 1 26 36 $\in$ orbit 1 26

and 1 2 14 ~ 58 49 conjugate with n, $\Rightarrow$ 116 57 5 ~ 1 26

Similarly,

N 1 2 43 113 43 = e conjugate with n in N

$\Rightarrow$ N 45 46 1 26 1 = e

N 1 26 59 65 30 = e conjugate with n in N

$\Rightarrow$ N 64 17 6 26 1 = e

N 1 26 94 43 40 = e conjugate with n in N

$\Rightarrow$ N 30 83 32 26 1 = e

N 1 6 96 55 30 = e conjugate with n in N

$\Rightarrow$ N 6 107 48 26 1 = e

N 1 26 30 32 78 = e conjugate with n in N

$\Rightarrow$ N 13 70 56 26 1 = e

N 1 26 66 43 40 = e conjugate with n in N

$\Rightarrow$ N 30 83 86 26 1 = e

N 1 2 23 51 83 = e conjugate with n in N

$\Rightarrow$ N 59 104 2 26 1 = e

N 1 6 98 69 36 = e conjugate with n in N

$\Rightarrow$ N 16 119 7 26 1 = e

N 1 6 58 50 15 = e conjugate with n in N

$\Rightarrow$ N 40 68 8 26 1 = e

N 1 26 35 51 8 = e conjugate with n in N

$\Rightarrow$ N 11 70 9 26 1 = e

N 1 26 28 74 40 = e conjugate with n in N

$\Rightarrow$ N 28 39 13 26 1 = e

N 1 6 80 58 69 = e conjugate with n in N

$\Rightarrow$ N 60 66 14 26 1 = e

N 1 6 38 100 8 = e conjugate with n in N

$\Rightarrow$ N 21 18 16 26 1 = e

N 1 6 61 50 37 = e conjugate with n in N

$\Rightarrow$ N 15 64 17 26 1 = e

N 1 26 54 1 65 = e conjugate with n in N

$\Rightarrow$ N 26 79 21 26 1 = e

N 1 26 48 9 75 = e conjugate with n in N

$\Rightarrow$ N 52 43 24 26 1 = e

N 1 26 36 34 30 = e conjugate with n in N

$\Rightarrow$ N 28 45 29 26 1 = e

N 1 26 47 79 16 = e conjugate with n in N

$\Rightarrow$ N 64 63 36 26 1 = e

N 1 6 46 13 37 = e conjugate with n in N

$\Rightarrow$ N 28 32 39 26 1 = e

N 1 6 109 33 75 = e conjugate with n in N

$\Rightarrow$ N 51 76 42 26 1 = e

Thus,

1 26 1 ~ 45 46 $\Rightarrow$ 1 26 1 $\in$ orbit 1 2

1 26 6 ~ 64 17 $\Rightarrow$ 1 26 6 $\in$ orbit 1 26

1 26 32 ~ 30 83 $\Rightarrow$ 1 26 32 $\in$ orbit 1 26

1 26 48 ~ 6 107 $\Rightarrow$ 1 26 48 $\in$ orbit 1 6

1 26 56 ~ 13 70 $\Rightarrow$ 1 26 56 $\in$ orbit 1 26

$1\ 26\ 86 \sim 30\ 83 \Rightarrow 1\ 26\ 86 \in$ orbit $1\ 26$

$1\ 26\ 2 \sim 59\ 104 \Rightarrow 1\ 26\ 2 \in$ orbit $1\ 2$

$1\ 26\ 7 \sim 16\ 119 \Rightarrow 1\ 26\ 7 \in$ orbit $1\ 6$

$1\ 26\ 8 \sim 40\ 68 \Rightarrow 1\ 26\ 8 \in$ orbit $1\ 6$

$1\ 26\ 9 \sim 11\ 70 \Rightarrow 1\ 26\ 9 \in$ orbit $1\ 26$

$1\ 26\ 13 \sim 28\ 39 \Rightarrow 1\ 26\ 13 \in$ orbit $1\ 26$

$1\ 26\ 14 \sim 60\ 66 \Rightarrow 1\ 26\ 14 \in$ orbit $1\ 6$

$1\ 26\ 16 \sim 21\ 18 \Rightarrow 1\ 26\ 16 \in$ orbit $1\ 6$

$1\ 26\ 21 \sim 26\ 79 \Rightarrow 1\ 26\ 21 \in$ orbit $1\ 26$

$1\ 26\ 24 \sim 52\ 43 \Rightarrow 1\ 26\ 24 \in$ orbit $1\ 26$

$1\ 26\ 29 \sim 28\ 45 \Rightarrow 1\ 26\ 29 \in$ orbit $1\ 26$

$1\ 26\ 39 \sim 28\ 32 \Rightarrow 1\ 26\ 39 \in$ orbit $1\ 6$

$1\ 26\ 42 \sim 51\ 76 \Rightarrow 1\ 26\ 42 \in$ orbit $1\ 6$

Similarly,

$1\ 2\ 54 \sim 69\ 49\ 13 \sim 91\ 30\ 19 \sim 34\ 99\ 39 \sim 103\ 59\ 50 \sim 100$
$108\ 54 \sim 118\ 6\ 55 \sim 85\ 96\ 72 \sim 17\ 4\ 78 \sim 76\ 106\ 98 \sim 93\ 117$
$104 \sim 51\ 12\ 113$

$1\ 2\ 54 \sim 1\ 53\ 92 \in [1\ 26\ 92]$

$1\ 2\ 54 \sim 13\ 97\ 12 \in [1\ 26\ 12]$

$1\ 2\ 54 \sim 33\ 49\ 11 \in [1\ 6\ 11]$

$1\ 2\ 54 \sim 84\ 9\ 23 \in [1\ 6\ 23]$

$1\ 2\ 54\ 14 \sim 16\ 6\ 54 \in [1\ 2\ 54]$

Since $1\ 6 = q_1\ 1\ 40 \Rightarrow 6\ 1\ 6 = 6\ q_1\ 1\ 40$

$\Rightarrow$ 6 1 6 = q_1 40 1 40

Conjugate with n in N, we have: $(6\ 1\ 6)^n = (q_1\ 40\ 1\ 40)^n$

$\Rightarrow$ 1 6 1 = $(q_1)^n$ 41 6 41 and 1 6 1 = q_1 1 $\underline{40\ 1}$

$\Rightarrow$ 1 6 1 = q_1 1 q_1 40 41 (since 41 1 = q_1 6 40 $\Rightarrow$ 40 1 = q_1 40 41) $\Rightarrow$ 1 6 1 = q_1 q_1 40 41 40

Similarly,

6 1 6 = 6 q_1 1 40 $\Rightarrow$ 6 1 6 = q_1 40 1 40

$\Rightarrow$ 6 1 6 = q_1 40 1 40 = q_1 40 41 40

Conjugate with n in N, we have:

41 6 41 = $(q_1)^n$ 1 6 1 = $(q_1)^n$ 40 1 40

$\Rightarrow$ 1 6 1 ~ 41 6 41 ~ 40 41 40 ~ 40 1 40 ~ 6 1 6

Continuing conjugate with n in N, we obtain:

1 6 1 ~ 1 40 1 ~ 6 1 6 ~ 6 41 6 ~ 40 1 40 ~ 40 41 40 ~ 41 6 41 ~ 41 40 41 ~ 49 113 49 ~ 49 120 49 ~ 50 71 50 ~ 50 103 50 ~ 63 70 63 ~ 63 96 63 ~ 65 75 65 ~ 65 95 65 ~ 67 83 67 ~ 67 117 67 ~ 70 63 70 ~ 70 112 70 ~ 71 50 71 ~ 71 73 71 ~ 73 71 73 ~ 73 103 73 ~ 75 65 75 ~ 75 76 75 ~ 76 75 76 ~ 76 95 76 ~ 81 113 81 ~ 83 67 83 ~ 86 83 86 ~ 95 65 95 ~ 96 63 96 ~ 103 50 103 ~ 112 70 112 ~ 113 49 113 ~ 117 67 117 ~ 120 49 120

Similarly,

1 26 3 ~ 62 70 1 ~ 3 117 1 ~ 45 119 1 ~ 24 29 4 ~ 45 3 6 ~ 109 33 7 ~ 34 12 8 ~ 69 105 9 ~ 41 31 10 ~ 41 64 11 ~ 37 8

12 ~ 75 117 13 ~ 72 4 15 ~ 119 111 17 ~ 91 11 18 ~ 36 85 19 ~ 14 23 20 ~ 39 102 21 ~ 110 21 23 ~ 108 116 25 ~ 117 3 26 ~ 71 114 27 ~ 78 80 28 ~ 109 88 29 ~ 67 13 30 ~ 60 88 31 ~ 16 6 32 ~ 86 113 33 ~ 39 20 34 ~ 14 106 35 ~ 22 20 37 ~ 64 85 38 ~ 36 11 40 ~ 88 60 42 ~ 62 3 43 ~ 50 81 44 ~ 96 106 46 ~ 22 21 47 ~ 91 89 48 ~ 24 57 49 ~ 22 108 50 ~ 109 10 51 ~ 39 108 52 ~ 7 57 53 ~ 120 114 54 ~ 112 28 55 ~ 16 17 56 ~ 15 53 57 ~ 32 66 58 ~ 115 20 59 ~ 31 42 60 ~ 22 12 61 ~ 21 110 63 ~ 94 42 64 ~ 5 106 65 ~ 107 13 66 ~ 45 92 67 ~ 2 27 68 ~ 78 82 70 ~ 48 29 73 ~ 72 79 73 ~ 16 98 74 ~ 75 32 76 ~ 72 85 77 ~ 109 53 79 ~ 13 67 80 ~ 52 44 81 ~ 66 32 82 ~ 72 51 83 ~ 75 111 84 ~ 113 38 85 ~ 61 114 87 ~ 94 15 88 ~ 41 104 89 ~ 62 17 90 ~ 69 111 92 ~ 19 4 93 ~ 106 96 95 ~ 118 34 96 ~ 91 113 97 ~ 9 28 98 ~ 45 105 99 ~ 75 9 100 ~ 43 105 101 ~ 71 12 102 ~ 94 83 103 ~ 86 51 104 ~ 16 26 105 ~ 71 110 108 ~ 14 61 110 ~ 107 99 111 ~ 72 42 113 ~ 102 87 114 ~ 5 21 115 ~ 120 63 116 ~ 26 1 117

Table 6.1. The Double Coset $N\omega N = [\omega]$,

where $N = L_2(16){:}\,4$, $\omega = \{1,2,\ldots,120\}$

$[\omega]$	Coset Stabilising subgroup $N^{(\omega)}$	Number of cosets in $[\omega]$

[*]	N is transitive on ω = {1,2 ..., 120} |N| = 16320	1
[1]	$N^{(1)} = N^1 \cong \langle$ (2, 3, 5, 11, 12, 27, 25, 24, 10, 7, 18, 38, 41, 20, 21, 8, 4)(6, 14, 28, 54, 64, 100, 56, 55, 29, 23, 9, 22, 43, 78, 72, 36, 16)(13, 19, 40, 74, 37, 32, 62, 108, 84, 47, 45, 66, 34, 17, 15, 33, 30)(26, 50, 88, 97, 73, 99, 119, 114, 118, 120, 102, 70, 98, 113, 77, 42, 52)(31, 58, 96, 53, 68, 111, 110, 94, 117, 109, 65, 76, 115, 83, 46, 82, 60)(35, 67, 81, 44, 80, 63, 107, 87, 49, 86, 93, 92, 51, 91, 89, 61, 69)(39, 48, 85, 71, 112, 103, 57, 101, 116, 95, 90, 106, 59, 105, 104, 79, 75), (3, 5, 12, 10, 4, 8, 20, 7)(6, 15, 28, 19, 9, 13, 29, 17)(11, 25, 41, 24, 21, 38, 27, 18)(14, 30, 56, 47, 23, 33, 64, 32)(16, 34, 36, 45, 22, 40, 43, 37)(26, 51, 76, 39, 42, 49, 53, 48)(31, 59, 99, 63, 46, 57, 102, 61)(35, 68, 71, 73, 44, 65, 79, 70)(50, 89, 82, 116, 77, 107, 60, 90)(52, 93, 94, 95)(54, 74, 72, 84, 55, 66, 78, 62)(58, 104, 120, 91, 83, 112, 119, 87)(67, 110, 101, 98, 81, 117, 106, 97)(69, 96, 75, 113, 80, 115, 85, 88)(86, 111, 103, 118, 92, 109, 105, 114)(100, 108) $\rangle$, $\|N^{(1)}\|$ = 136 $N^{(1)}$ has orbits: { 1 }, { 2, 3, 4, 5, 7, 8, 10, 11, 12, 18, 20, 21, 24, 25, 27, 38, 41 }, { 6, 9, 13, 14, 15, 16, 17, 19, 22, 23, 28, 29, 30, 32, 33, 34, 36, 37, 40, 43, 45, 47, 54, 55, 56, 62,64, 66, 72, 74, 78, 84, 100, 108 }, { 26, 31, 35, 39, 42, 44, 46, 48, 49, 50, 51, 52, 53, 57, 58, 59, 60, 61, 63, 65, 67, 68, 69, 70, 71, 73, 75, 76, 77, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99,	120

	101, 102,103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120 } on ω (d^5)70 23 51 2 1 = e = $((d^5)^{p8})$1 2 51 45 46 $\Rightarrow$ $((d^5)^{p8})$ 70 23 45 46 = e = p_1^{p45} 1 2 108 100 $\Rightarrow$1 2 = $(p2)^{p59}$ 2 1 = $((p_1)^{p45})^{-1}$ 100 108 = $((p_1)^{p46})^{-1}$ 108 100 $\Rightarrow$ 1 2 ~ 2 1 ~ 100 108 ~ 108 100 $((d^5)^{p8})^{q22}$ $(70\ 23\ 45\ 46)^{q22}$ = e = (q_1)40 6 1 41 $\Rightarrow$ 1 6 = (q_1)1 40 $\Rightarrow$ 1 6 ~ 1 40 41 100 1 2 = a_3 26 100 $\Rightarrow$ $\underline{2}$ 41 100 1 2 $\underline{107}$ = $\underline{2}$ a_3 26 100 $\underline{107}$ $\Rightarrow$ $\bar{2}$ 41 100 1 2 $\overline{107}$ = a_3 1 26 100 $\overline{107}$ $\Rightarrow$ $(j_1)^{-1}$ = a_3 1 26 100 107 $\Rightarrow$ e = $(j_1)a_3$ 1 26 100 107 and e = $((j_1)a_3)^{j4}$ 1 26 55 69 $\Rightarrow$ 1 26 = r_1 69 55 = r_2 107 100.) $\Rightarrow$ 1 26 ~ 69 55 ~ 107 100	
[1 2] Since [1 2 1] = [1] [1 2 52] = [1 6 52] [1 2 51] = [1 2] [1 2 50] = [1 26] [1 2 3] = [1 6] [1 2 54] = [1 26 31]	$\|N^{(1\ 2)}\|$ = 32 and orbits of $N^{(1\ 2)}$ on ω are: { 1, 2, 100, 108 }, { 52, 93, 94, 95 }, { 11, 18, 21, 24, 25, 26, 27, 38, 39, 41, 42, 48, 49, 51, 53, 76 }, { 50, 58, 60, 77, 82, 83, 87, 89, 90, 91, 104, 107,112, 116, 119, 120 }, { 54, 55, 62, 66, 67, 72, 74, 78, 81, 84, 97, 98, 101, 106, 110, 117 }, { 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 15, 17, 19, 20, 28, 29, 69, 75, 80, 85, 86, 88, 92, 96, 103, 105, 109,111, 113, 114, 115,118 }, and { 14, 16, 22, 23, 30, 31, 32, 33, 34, 35, 36, 37, 40, 43, 44, 45, 46, 47, 56, 57, 59, 61, 63, 64, 65, 68, 70, 71, 73, 79, 99, 102 } 1 2 52 = q_{70} 94 93 $\in$ 1 6 1 2 51 = d^5 70 23 $\in$ 1 2 1 2 50 = r_{70} 48 13 $\in$ 1 26 1 2 3 = q_{71} 104 62 $\in$ 1 6	510

	1 2 14 = r_{71} 58 49 ∈ 1 26	
[1 6]	$\lvert N^{(1\ 6)}\rvert$ = 8 and orbits of $N^{(1\ 6)}$ on ω are: { 1 }, { 41 }, { 6, 40 }, { 63, 70, 96, 112 }, { 2, 3, 5, 7, 8, 24, 25, 27 }, { 4, 10, 11, 12, 18, 20, 21, 38 }, { 9, 15, 22, 30, 32, 55, 56, 108 }, { 13, 14, 16, 37, 45, 64, 66, 78 }, { 17, 34, 43, 47, 54, 72, 84, 100 }, { 19, 23, 28, 29, 33, 36, 62, 74 }, { 26, 35, 48, 93, 99, 101, 109, 115 }, { 31, 39, 68, 80, 88, 97, 107, 116 }, { 42, 53, 57, 58, 85, 89, 91, 114 }, { 44, 59, 82, 87, 98, 102, 105, 110 }, { 46, 69, 77, 92, 94, 104, 106, 118 }, { 49, 50, 73, 75, 81, 83, 95, 117 }, { 51, 52, 60, 61, 79, 90, 111,119 }, { 65, 67, 71, 76, 86, 103, 113, 120 } 1 6 41 ~ 63 112 ∈ 1 2 1 6 63 ~ 99 66 ∈ 1 26 1 6 2 ~ 30 89 ∈ 1 6 1 6 4 ~ 108 15 ∈ 1 26 1 6 9 ~ 13 10 ∈ 1 26 1 6 13 ~ 45 67 ∈ 1 6 1 6 17 ~ 43 36 ∈ 1 26 1 6 19 ~ 39 7 ∈ 1 26 1 6 26 ~ 110 70 ∈ 1 26 1 6 31 ~ 112 44 ∈ 1 26 1 6 44 ~ 89 17 ∈ 1 26 1 6 49 ~72 96 ∈ 1 6 1 6 51 ~ 55 84 ∈ 1 6 1 6 65 ~ 59 90 ∈ 1 2	2040
[1 26]	$\lvert N^{(1\ 26)}\rvert$ = 6 and orbits of $N^{(1\ 26)}$ on ω are: { 1, 69, 107 }, { 3, 34, 37 }, { 6, 43, 84 }, { 26, 55, 100 }, { 32, 66, 92 }, { 48, 85, 117 },	2720

	{ 56, 59, 116 }, { 86, 90, 91 }, { 2, 5, 46, 51, 93, 96 }, { 4, 11, 23, 49, 54, 89 }, { 7, 18, 35, 61, 63, 87 }, { 8, 10, 12, 15, 38, 40 }, { 9, 19, 28, 33, 67, 80 }, { 13, 30, 65, 70, 110, 119 }, { 14, 22, 75, 98, 99, 112 }, { 16, 20, 25, 78, 79, 103 }, { 17, 45, 53, 62, 74, 83 }, { 21, 27, 47, 73, 108, 113 }, { 24, 41, 68, 77, 97, 115 }, { 29, 58, 64, 82, 95, 106 }, { 31, 50, 52, 60, 101, 105 }, { 36, 72, 76, 102, 111, 114 }, { 39, 44, 71, 81, 94, 109 }, { 42, 57, 88, 104, 118, 120 } 1 26 1 ~ 45 46 ∈ 1 2 1 26 6 ~ 64 17 ∈ 1 26 1 26 32 ~ 30 83 ∈ 1 26 1 26 48 ~ 6 107 ∈ 1 6 1 26 56 ~ 13 70 ∈ 1 26 1 26 86 ~ 30 83 ∈ 1 26 1 26 2 ~ 59 104 ∈ 1 2 1 26 7 ~ 16 119 ∈ 1 6 1 26 8 ~ 40 68 ∈ 1 6 1 26 9 ~ 11 70 ∈ 1 26 1 26 13 ~ 28 39 ∈ 1 26 1 26 14 ~ 60 66 ∈ 1 6 1 26 16 ~ 21 18 ∈ 1 6 1 26 17 ~ 15 64 ∈ 1 6 1 26 21 ~ 26 79 ∈ 1 26 1 26 24 ~ 52 43 ∈ 1 26 1 26 29 ~ 28 45 ∈ 1 26 1 26 36 ~ 64 63 ∈ 1 26 1 26 39 ~ 28 32 ∈ 1 6 1 26 42 ~ 51 76 ∈ 1 6	
[1 2 54]	$N^{1\ 2\ 54} = \langle e \rangle$. But 1 2 51 ~ 70 23, 1 2 23 ~ 83 51, 1 2 52 ~ 94 93, 1 2 50 ~ 48 13, 1 2 3 ~ 104 62. Now $\lvert N^{(1\ 2\ 54)} \rvert = 24$ and $N^{(1\ 2\ 54)}$ is	680

	transitive on ω	
[1 6 1]	$N^{1\ 6\ 1} = \langle e \rangle$. But 1 6 1 ~ 6 1 6 ~ 40 1 40 ~ 41 6 41 ~ 49 120 49 ~ 50 71 50 ~ 63 70 63 ~ 65 75 65 ~ 67 83 67 ~ 70 63 70 ~ 71 73 71 ~ 73 71 73 ~ 75 65 75 ~ 76 95 76 ~ 81 113 81 ~ 83 67 83 ~ 86 83 86 ~ 95 65 95 ~ 96 63 96 ~ 103 50 103 ~ 112 70 112 ~ 113 49 113 ~ 117 67 117 ~ 120 49 120, also 1 26 3 ~ 62 70 1 ~ 3 117 1 ~ 45 119 1 ~ 24 29 4 ~ 45 3 6 ~ 109 33 7 ~ 34 12 8 ~ 69 105 9 ~ 41 31 10 ~ 41 64 11 ~ 37 8 12 ~ 75 117 13 ~ 72 4 15 ~ 119 111 17 ~ 91 11 18 ~ 36 85 19 ~ 14 23 20 ~ 39 102 21 ~ 110 21 23 ~ 108 116 25 ~ 117 3 26 ~ 71 114 27 ~ 78 80 28 ~ 109 88 29 ~ 67 13 30 ~ 60 88 31 ~ 16 6 32 ~ 86 113 33 ~ 39 20 34 ~ 14 106 35 ~ 22 20 37 ~ 64 85 38 ~ 36 11 40 ~ 88 60 42 ~ 62 3 43 ~ 50 81 44 ~ 96 106 46 ~ 22 21 47 ~ 91 89 48 ~ 24 57 49 ~ 22 108 50 ~ 109 10 51 ~ 39 108 52 ~ 7 57 53 ~ 120 114 54 ~ 112 28 55 ~ 16 17 56 ~ 15 53 57 ~ 32 66 58 ~ 115 20 59 ~ 31 42 60 ~ 22 12 61 ~ 21 110 63 ~ 94 42 64 ~ 5 106 65 ~ 107 13 66 ~ 45 92 67 ~ 2 27 68 ~ 78 82 70 ~ 48 29 73 ~ 72 79 73 ~ 16 98 74 ~ 75 32 76 ~ 72 85 77 ~ 109 53 79 ~ 13 67 80 ~ 52 44 81 ~ 66 32 82 ~ 72 51 83 ~ 75 111 84 ~ 113 38 85 ~ 61 114 87 ~ 94 15 88 ~ 41 104 89 ~ 62 17 90 ~ 69 111 92 ~ 19 4 93 ~ 106 96 95 ~ 118 34 96 ~ 91 113 97 ~ 9 28 98 ~ 45 105 99 ~ 75 9 100 ~ 43 105 101 ~ 71 12 102 ~ 94 83 103 ~ 86 51 104 ~ 16 26 105 ~ 71 110 108 ~ 14 61 110 ~ 107 99 111 ~ 72 42 113 ~ 102 87 114 ~ 5 21 115 ~ 120 63 116 ~ 26 1 117 ~ 62 9 119 Now $\lvert N^{(1\ 6\ 1)} \rvert = 192$ and $N^{(1\ 6\ 1)}$ is transitive on ω	85

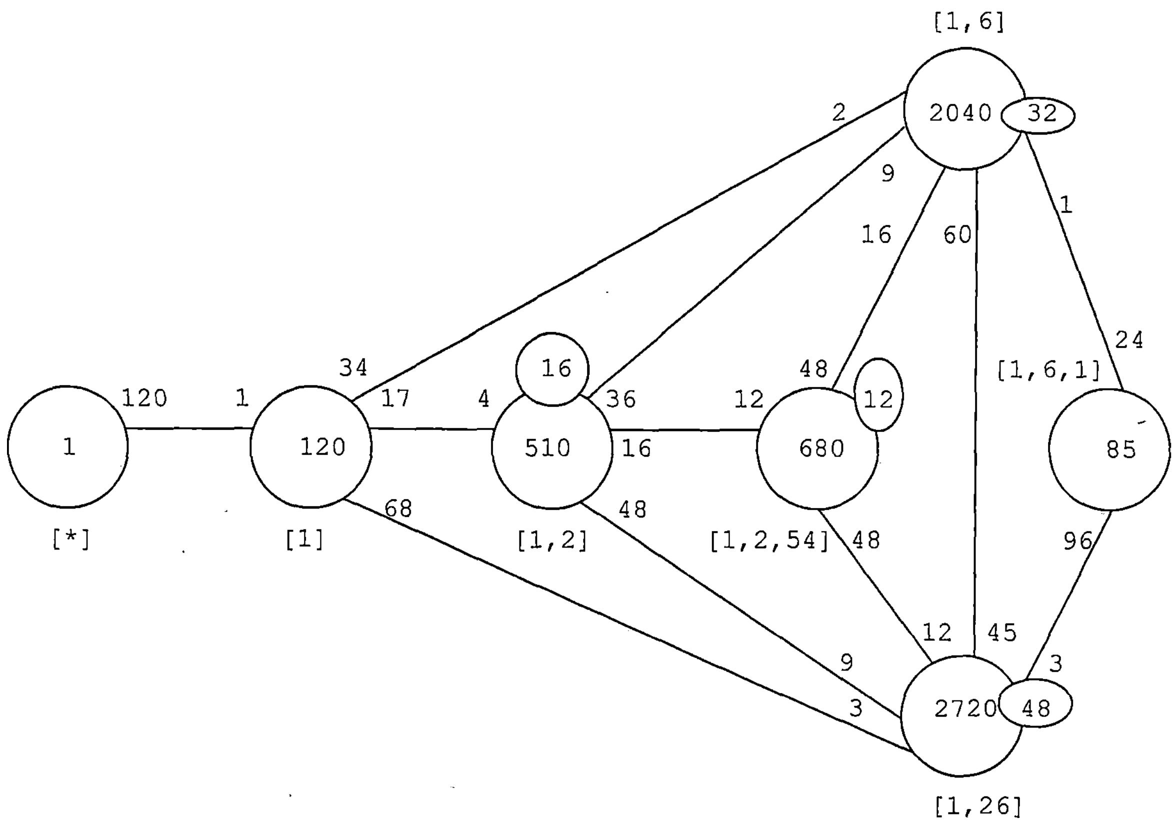

Figure 6.1. Cayley Graph of J_3: 2 over $L_2(16)$: 4

The Identification of Cosets Labeling

MAGMA Labeling-Single coset

2 [1]	10 [7]
3 [2]	12 [8]
4 [3]	13 [9]
6 [4]	14 [10]
7 [5]	15 [11]
8 [6]	16 [12]

17 [13]

19 [14]

20 [15]

22 [16]

23 [17]

26 [18]

27 [19]

28 [20]

32 [21]

33 [22]

35 [23]

36 [24]

37 [25]

38 [26]

40 [27]

41 [28]

43 [29]

45 [30]

47 [31]

49 [32]

50 [33]

55 [34]

56 [35]

57 [36]

58 [37]

67 [38]

68 [39]

69 [40]

71 [41]

77 [42]

78 [43]

79 [44]

81 [45]

84 [46]

85 [47]

86 [48]

87 [49]

89 [50]

90 [51]

92 [52]

94 [53]

96 [54]

100 [55]

103 [56]

104 [57]

109 [58]

110 [59]

112 [60]

113 [61]

116 [62]

117 [63]

118 [64]

130 [65]

131 [66]

132 [67]

133 [68]

135 [69]

136 [70]

137 [71]

138 [72]

139 [73]

140 [74]

165 [75]

166 [76]

181 [77]

182 [78]

183 [79]

185 [80]

187 [81]

194 [82]

196 [83]

197 [84]

198 [85]

200 [86]

202 [87]

204 [88]

205 [89]

207 [90]

208 [91]

210 [92]

213 [93]

214 [94]

215 [95]

221 [96]

223 [97]

231 [98]

237 [99]

238 [100]

239 [101]

240 [102]

242 [103]

255 [104]

257 [105]	379 [113]
259 [106]	380 [114]
263 [107]	381 [115]
272 [108]	412 [116]
304 [109]	413 [117]
305 [110]	438 [118]
307 [111]	444 [119]
312 [112]	453 [120]

The group is defined by the symmetric presentation. Its index is at most:

$$\frac{|N|}{|N|}+\frac{|N|}{|N^{(1)}|}+\frac{|N|}{|N^{(1,2)}|}+\frac{|N|}{|N^{(1,6)}|}+\frac{|N|}{|N^{(1,26)}|}+\frac{|N|}{|N^{(1,2,54)}|}+\frac{|N|}{|N^{(1,6,1)}|}$$

= 1 + 120 + 510 + 2040 + 2720 + 680 + 85 = 6156

(According to the J_3: 2 over $L_2(16)$: 4 graph)

The index of N in G has at most 6156. $\Rightarrow$ the order of G is at most 6156 |N| = 6156 x 16320 = 100465920. (G has order at most $|L_2(16): 4|(6156)$ = 6156 x 16320 = 100465920)

Thus, the order of G is most at 100465920.

Since the group J_3: 2 is generated by x, y, z, and t

$\Rightarrow J_3: 2$ is an image of G $\Rightarrow |G| \geq |J_3: 2|$

$\Rightarrow |G| \geq 100465920 \Rightarrow 100465920 \leq |G| \leq 100465920$

$\Rightarrow |G| = 100465920 \Rightarrow G \cong J_3: 2$.

The smallest index of any subgroup in J_3: 2 is 6156.

Thus, J_3: 2 is usually represented as permutation group on 6156 letters. Now, from our Cayley graph,

$J_3: 2 = NeN \cup Nt_1N \cup Nt_1t_2N \cup Nt_1t_6N \cup Nt_1t_{26}N \cup Nt_1t_2t_{54}N \cup Nt_1t_6t_1N$, where $N = L_2(16): 4$.

We see that every element of J_3: 2 can be written as permutation of $L_2(16)$: 4, on 120 letters, followed by a word of length at most 3 in the symmetric generators $\{t_1, t_2, .., t_{120}\}$. This representation of J_3: 2 is much more convenient to work with that its usual permutation representation.

However, we can improve this representation as follow: $L_2(16)$ can be represented as a group of 2 x 2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where determinant: ad - bc = 1, or equivalently a square in GF(16). Then J_3: 2 is isomorphic to $L_2(16)$ extended by the field automorphism $x \rightarrow x^2$ for all $x \in$ GF(16). Hence elements of J_3: 2 can be represented as triples (A,m,w), where A is a 2 x 2 matrix, m: $0 \leq m \leq 3$, is the power of the automorphism, and w is a word of length at most 3, in terms of the symmetric generators $t_1, t_2, .., t_{120}$. Although we have not done this here, a computer program can be worked with the elements of J_3: 2 represented in this manner.

APPENDIX A

S_7

MAGMA works for symmetric group of S_7 :

$(1\ 2\ 4)(3\ 6\ 5) = (xy)^{x^5yx}$

$(2\ 6)(4\ 5) = y$

$(1\ 3)(2\ 4\ 6\ 5) = (xyx^3yx)^{x^6yx^2}$

$(1\ 3\ 4\ 7\ 6\ 5\ 2) = x^{2^{xyxyx}}$

$(2\ 5)(4\ 6) = y\, y^{x^3yx^5y}$

$(2\ 4)(5\ 6) = y^{x^3yx^5y}$

$(1\ 3\ 7)(2\ 5\ 4) = (xy)^{xyx^5y}$

$(1\ 7\ 3)(2\ 4\ 5) = (xy)^{x^4yxy}$

```
N7:=Stabiliser(N,7);
N71:= Stabiliser(N7,1);
for g in N do if 7^g eq 1 and 1^g eq 3 then
N71:=sub<N|N71,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 7 then
N71:=sub<N|N71,g>; end if; end for;
N712:=Stabiliser(N71,2);
for g in N do if 7^g eq 1 and 1^g eq 6 and 2^g eq 7 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 5 and 2^g eq 6 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 4 and 2^g eq 5 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 4 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 4 and 2^g eq 3 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 6 and 2^g eq 1 then
N712:=sub<N|N712,g>; end if; end for;
tra3 := Transversal(N, sub<N | N712>);
```

```
for i := 1 to #tra3 do
      ss := [7,1,2]^tra3[i];
      cst[prodim(1, ts, ss)] := ss;
    end for;
   m:=0;
 for i in [1..30] do if cst[i] ne [] then m:=m+1; end if;
end for;
…
print m;
29

Order (N);
168
Order (N7);
24
Order (N71);
12
Order (N712);
21
```

APPENDIX B

S_8

MAGMA works for symmetric group of S_8 :

```
N7:=Stabiliser(N,7);
N71:= Stabiliser(N7,1);
N717:=Stabiliser(N71,7);
for g in N do if 7^g eq 4 and 1^g eq 6 and 7^g eq 4 then
N717:=sub<N|N717,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 7^g eq 1 then
N717:=sub<N|N717,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 2 and 7^g eq 5 then
N717:=sub<N|N717,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 4 and 7^g eq 6 then
N717:=sub<N|N717,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 7^g eq 2 then
N717:=sub<N|N717,g>; end if; end for;
N7173:=Stabiliser(N717,3);
N713:=Stabiliser(N71,3);
N712:=Stabiliser(N71,2);
for g in N do if 7^g eq 7 and 1^g eq 5 and 2^g eq 1 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 5 then
N712:=sub<N|N712,g>; end if; end for;
N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 7 and 1^g eq 5 and 2^g eq 3 and 3^g
eq 4 then N7123:=sub<N|N7123,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 4 and 3^g
eq 2 then N7123:=sub<N|N7123,g>; end if; end for;
N7137:=Stabiliser(N713,7);
for g in N do if 7^g eq 1 and 1^g eq 3 and 3^g eq 7 and 7^g
eq 1 then N7137:=sub<N|N7137,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 7 and 3^g eq 1 and 7^g
eq 3 then N7137:=sub<N|N7137,g>; end if; end for;
N71237:=Stabiliser(N7123,7);
for g in N do if 7^g eq 1 and 1^g eq 3 and 2^g eq 6 and 3^g
eq 7 and 7^g eq 1  then N71237:=sub<N|N71237,g>; end if;
end for;
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 1 and 3^g
eq 6 and 7^g eq 2  then N71237:=sub<N|N71237,g>; end if;
end for;
for g in N do if 7^g eq 3 and 1^g eq 6 and 2^g eq 1 and 3^g
eq 4 and 7^g eq 3  then N71237:=sub<N|N71237,g>; end if;
end for;
for g in N do if 7^g eq 4 and 1^g eq 7 and 2^g eq 6 and 3^g
eq 5 and 7^g eq 4  then N71237:=sub<N|N71237,g>; end if;
end for;
```

```
for g in N do if 7^g eq 5 and 1^g eq 6 and 2^g eq 2 and 3^g
eq 1 and 7^g eq 5  then N71237:=sub<N|N71237,g>; end if;
end for;
for g in N do if 7^g eq 6 and 1^g eq 2 and 2^g eq 5 and 3^g
eq 7 and 7^g eq 6  then N71237:=sub<N|N71237,g>; end if;
end for;
tra7 := Transversal(N, sub<N | N71237>);
for i := 1 to #tra7 do
      ss := [7,1,2,3,7]^tra7[i];
      cst[prodim(1, ts, ss)] := ss;
    end for;
   m:=0;
 for i in [1..240] do if cst[i] ne [] then m:=m+1; end if;
end for;
…
print m;
239

Order (N);
168
Order (N7);
24
Order (N71);
4
Order (N717);
24
Order (N712);
3
Order (N713);
4
Order (N7173);
24
Order (N7123);
3
Order (N7137);
12
Order (N71237);
21
```

APPENDIX C

$3^{\bullet}S_7$

MAGMA works for symmetric group of $3^{\cdot}S_7$:

```
N7:=Stabiliser(N,7);
N71:= Stabiliser(N7,1);
N717:=Stabiliser(N71,7);
for g in N do if 7^g eq 4 and 1^g eq 6 and 7^g eq 4 then
N717:=sub<N|N717,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 7^g eq 1 then
N717:=sub<N|N717,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 2 and 7^g eq 5then
N717:=sub<N|N717,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 4 and 7^g eq 6 then
N717:=sub<N|N717,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 7^g eq 2 then
N717:=sub<N|N717,g>; end if; end for;
N7173:=Stabiliser(N717,3);
for g in N do if 7^g eq 7 and 1^g eq 3 and 7^g eq 7 and 3^g
eq 1 then N7173:=sub<N|N7173,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 7^g eq 7 and 3^g
eq 2 then N7173:=sub<N|N7173,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 1 and 7^g eq 2 and 3^g
eq 4 then N7173:=sub<N|N7173,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 4 and 7^g eq 7 and 3^g
eq 5 then N7173:=sub<N|N7173,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 2 and 7^g eq 7 and 3^g
eq 6 then N7173:=sub<N|N7173,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 3 and 7^g eq 1 and 3^g
eq 7 then N7173:=sub<N|N7173,g>; end if; end for;
N713:=Stabiliser(N71,3);
for g in N do if 7^g eq 6 and 1^g eq 1 and 3^g eq 5 then
N713:=sub<N|N713,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 1 and 3^g eq 2 then
N713:=sub<N|N713,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 1 and 3^g eq 6 then
N713:=sub<N|N713,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 1 and 3^g eq 4 then
N713:=sub<N|N713,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 1 and 3^g eq 7 then
N713:=sub<N|N713,g>; end if; end for;
N7131:=Stabiliser(N713,1);
for g in N do if 7^g eq 5 and 1^g eq 2 and 3^g eq 3 and 1^g
eq 2 then N7131:=sub<N|N7131,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 3 and 3^g eq 7 and 1^g
eq 3 then N7131:=sub<N|N7131,g>; end if; end for;
```

```
for g in N do if 7^g eq 1 and 1^g eq 4 and 3^g eq 2 and 1^g
eq 4 then N7131:=sub<N|N7131,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 5 and 3^g eq 1 and 1^g
eq 5 then N7131:=sub<N|N7131,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 6 and 3^g eq 1 and 1^g
eq 6 then N7131:=sub<N|N7131,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 7 and 3^g eq 4 and 1^g
eq 7 then N7131:=sub<N|N7131,g>; end if; end for;
N712:=Stabiliser(N71,2);
for g in N do if 7^g eq 6 and 1^g eq 3 and 2^g eq 1 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 4 and 2^g eq 3 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 4 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 2 and 2^g eq 5 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 5 and 2^g eq 6 then
N712:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 6 and 2^g eq 7 then
N712:=sub<N|N712,g>; end if; end for;

Order (N);
168
Order (N7);
24
Order (N71);
4
Order (N717);
24
Order (N712);
7
Order (N713);
24
Order (N7173);
168
Order (N7131);
168
```

APPENDIX D

$2^{3_1}\, 2^{3_1+3_2} : L_3(2)$

MAGMA works for symmetric group of $2^{3_1} 2^{3_1+3_2} : L_3(2)$:

```
N7:=Stabiliser(N,7);
N71:= Stabiliser(N7,1);
N713:=Stabiliser(N71,3);
for g in N do if 7^g eq 3 and 1^g eq 7 and 3^g eq 1 then
N713:=sub<N|N713,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 3 and 3^g eq 7 then
N713:=sub<N|N713,g>; end if; end for;
N7132:=Stabiliser(N713,2);
N717:=Stabiliser(N71,7);
for g in N do if 7^g eq 3 and 1^g eq 1 and 7^g eq 3 then
N717:=sub<N|N717,g>; end if; end for;
N7171:=Stabiliser(N717,1);
for g in N do if 7^g eq 3 and 1^g eq 7 and 7^g eq 3 and 1^g
eq 7  then N7171:=sub<N|N7171,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 3 and 7^g eq 7 and 1^g
eq 3  then N7171:=sub<N|N7171,g>; end if; end for;
N712:=Stabiliser(N71,2);
N7121:=Stabiliser(N712,1);
for g in N do if 7^g eq 7 and 1^g eq 4 and 2^g eq 2  and
1^g eq 4 then N7121:=sub<N|N7121,g>; end if; end for;
N7127:=Stabiliser(N712,7);
for g in N do if 7^g eq 6 and 1^g eq 2 and 2^g eq 1  and
7^g eq 6 then N7127:=sub<N|N7127,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 2 and 2^g eq 1  and
7^g eq 3 then N7127:=sub<N|N7127,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 1 and 2^g eq 2  and
7^g eq 5 then N7127:=sub<N|N7127,g>; end if; end for;
N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 5 and 1^g eq 7 and 2^g eq 1  and
5^g eq 2 then N7125:=sub<N|N7125,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 7  and
5^g eq 1 then N7125:=sub<N|N7125,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 2 and 2^g eq 5  and
5^g eq 7 then N7125:=sub<N|N7125,g>; end if; end for;
N71271:=Stabiliser(N7127,1);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2  and
7^g eq 1 and 1^g eq 7 then N71271:=sub<N|N71271,g>; end if;
end for;
for g in N do if 7^g eq 7 and 1^g eq 5 and 2^g eq 2  and
7^g eq 7 and 1^g eq 5 then N71271:=sub<N|N71271,g>; end if;
end for;
tra9 := Transversal(N, sub<N | N71271>);
```

```
for i := 1 to #tra9 do
      ss := [7,1,2,7,1]^tra9[i];
      cst[prodim(1, ts, ss)] := ss;
    end for;
   m:=0;
for i in [1..512] do if cst[i] ne [] then m:=m+1; end if;
end for;
…
print m;
511

Order (N);
168
Order (N7);
24
Order (N71);
4
Order (N713);
12
Order (N7132);
3
Order (N717);
8
Order (N7171);
24
Order (N712);
1
Order (N7121);
2
Order (N7127);
4
Order (N7125);
4
Order (N71271);
6
```

APPENDIX E

J_3: 2

MAGMA works for symmetric group of J_3 :

PERMUTATION :

d^5 ~ (1, 89, 79, 23, 88, 2, 34, 21, 70, 47, 51, 99)(3, 22, 76, 60, 16, 19, 101, 33, 118, 10, 44, 50)(4, 63, 39, 112, 100, 29, 41, 45, 54, 7, 65, 28)(5, 13, 69, 66, 42, 48, 117, 72, 20, 107, 120, 24)(6, 9, 86, 77, 91, 67, 106, 27, 25, 105, 103, 119)(8, 98, 116, 26, 80, 15, 38, 55, 74, 83, 12, 49)(11, 115, 82, 110, 75, 53, 90, 104, 96, 92, 84, 85)(14, 58, 64, 68, 113, 61, 43, 81, 93, 95, 78, 73)(17, 97, 31, 114, 111, 102, 87, 18, 32, 62, 109, 52)(30, 36, 40, 57, 108, 71, 59, 46, 94, 35, 56, 37)

p_8 ~ (1, 46, 108, 70)(2, 45, 100, 23)(3, 9, 85, 114)(4, 6, 75, 118)(5, 82, 86, 91)(7, 48, 103, 39)(8, 104, 92, 77)(10, 55, 105, 54)(11, 69, 42, 20)(12, 67, 80, 74)(13, 87, 88, 60)(14, 94, 37, 93)(15, 50, 113, 112)(16, 68, 57, 99)(17, 97, 115, 98)(18, 96, 24, 19)(21, 109, 26, 29)(22, 34, 59, 64)(25, 72)(27, 83, 53, 116)(28, 81, 111, 66)(30, 40, 35, 56)(31, 95, 73, 52)(32, 47, 71, 79)(33, 65, 44, 102)(36, 61, 63, 43)(41, 119, 76, 89)(49, 110)(58, 101, 90, 62)(84, 120, 106, 107)

p_{45} ~ (1, 57, 63, 37, 94, 16, 71, 70)(2, 22, 36, 31, 52, 59, 32, 23)(3, 13, 58, 98, 6, 113, 119, 39)(4, 15, 89, 24, 9, 88, 90, 54)(5, 116, 18, 75, 92, 107, 55, 114)(7, 26, 17, 67, 115, 81, 103, 42)(8, 120, 97, 85, 86, 83, 48, 118)(10, 66, 19, 11, 96, 21, 105, 74)(12, 110, 53, 77, 20, 25, 84, 91)(14, 95, 44, 61, 46, 100, 33, 79)(27, 112, 28, 117, 106, 60, 29, 38)(30, 43, 45, 108, 35, 47, 73, 93)(34, 99)(40, 68, 102, 64)(41, 87, 69, 49, 101, 50, 80, 72)(51, 76, 82, 111, 78, 62, 104, 109)

p_{59} ~ (1, 100)(2, 108)(3, 118)(4, 114)(5, 88)(6, 85)(7, 96)(8, 113)(9, 75)(10, 115)(11, 21)(12, 111)(13, 86)(14, 73)(15, 92)(16, 30)(17, 105)(18, 48)(19, 103)(20, 109)(22, 33)(23, 70)(24, 39)(25, 49)(26, 42)(28, 80)(29, 69)(31, 37)(32, 36)(34, 102)(35, 57)(38, 51)(40, 99)(43, 47)(44, 59)(45, 46)(50, 104)(52, 93)(54, 98)(55, 97)(56, 68)(58, 90)(60, 91)(61, 79)(63, 71)(64, 65)(66, 74)(67, 81)(72, 110)(77, 112)(78, 117)(82, 87)(83, 116)(89, 119)(94, 95)(107, 120)

$(d-)p_1 \sim$ (1, 59, 52, 35)(2, 16, 94, 33)(3, 42, 4, 74)(5, 15, 88, 92)(6, 81, 9, 21)(7, 38, 103, 72)(8, 13, 113, 86)(10, 117, 105, 49)(11, 85, 67, 75)(12, 90, 28, 107)(17, 25, 115, 78)(18, 104, 97, 82)(19, 110, 96, 51)(20, 119, 29, 83)(22, 108, 30, 95)(23, 70)(24, 91, 98, 77)(26, 114, 66, 118)(27, 41, 62, 84)(32, 34, 79, 68)(36, 102, 61, 56)(39, 60, 54, 112)(40, 43, 65, 63)(44, 93, 57, 100)(45, 46)(47, 64, 71, 99)(48, 50, 55, 87)(53, 106, 101, 76)(58, 80, 120, 111)(69, 116, 109, 89)

$(d-)p_2 \sim$ (3, 4)(5, 8)(6, 9)(7, 10)(11, 21)(12, 20)(13, 15)(14, 23)(16, 22)(17, 19)(18, 24)(25, 38)(26, 42)(27, 41)(28, 29)(30, 33)(31, 46)(32, 47)(34, 40)(35, 44)(36, 43)(37, 45)(39, 48)(49, 51)(50, 77)(53, 76)(54, 55)(56, 64)(57, 59)(58, 83)(60, 82)(61, 63)(62, 84)(65, 68)(66, 74)(67, 81)(69, 80)(70, 73)(71, 79)(72, 78)(75, 85)(86, 92)(87, 91)(88, 113)(89, 107)(90, 116)(96, 115)(97, 98)(99, 102)(101, 106)(103, 105)(104, 112)(109, 111)(110, 117)(114, 118)(119, 120)

$q_{22} \sim$ (1, 22, 35, 90, 28, 53, 95, 45)(2, 101, 75, 18, 38, 82, 72, 110)(3, 77, 92, 73, 96, 98, 97, 20)(4, 118, 107, 36, 52, 56, 15, 50)(5, 117, 74, 116)(6, 47, 115, 62, 120, 114, 80, 23)(8, 67, 84, 65, 99, 16, 13, 103)(9, 34, 60, 68, 111, 29, 25, 57)(10, 89, 27, 113, 86, 71, 109, 87)(11, 17, 33, 37, 63, 78, 85, 21)(12, 58, 42, 39, 104, 54, 106, 81)(14, 70, 40, 26, 94, 48, 43, 64)(19, 102, 108, 66, 46, 41, 76, 49)(24, 88, 83, 59, 79, 30, 93, 51)(31, 112)(44, 55, 100, 61, 119, 105, 91, 69)

$q_{01} \sim$ (1, 51, 49, 9, 29, 13, 48, 42)(3, 67, 20, 65, 99, 57, 71, 59)(4, 106, 110, 5, 117, 68, 46, 70)(6, 19, 39, 17, 76, 28, 26, 53)(7, 44, 101, 31, 61, 79, 73, 108)(8, 12, 100, 35, 10, 102, 81, 98)(11, 74, 14, 18, 45, 89, 34, 112)(16, 40, 93, 64, 109, 72, 96, 77)(21, 107, 69, 114, 92, 95, 58, 43)(22, 60, 104, 54, 87, 105, 90, 85)(23, 30, 94, 111, 83, 116, 55, 38)(24, 120, 66, 103, 115, 86, 47, 78)(25, 80, 62, 50)(27, 84, 119, 91,52, 36, 56, 88)(32, 41, 33, 37, 113, 82, 118, 75)(63, 97)

$q_{02} \sim$ (1, 61, 5, 76, 88, 79, 100, 120, 114, 56, 112, 77, 68, 4, 107)(2, 80, 28, 108, 99, 105, 118, 57, 70, 84, 23, 35, 3, 17, 40)(6, 13, 96, 38, 24, 83, 33, 22, 116, 39, 51, 7, 86, 85, 106)(8, 11, 43, 64, 65, 47, 21, 113, 30, 97, 12, 62, 111, 55, 16)(9, 91, 63, 69, 49, 14, 73, 25, 29, 71, 60, 75, 54, 41, 98)(10, 66, 74, 115, 119, 82, 48, 31, 102, 53, 34, 37, 18, 87, 89)(15, 20, 104, 26, 72, 52, 103, 27, 19, 93, 110, 42, 50, 109, 92)(32, 44, 95, 94, 59, 36, 46, 81, 58, 78, 101, 117, 90, 67, 45)

q_{03} ~ (1, 68, 92, 48, 105, 93, 36, 102, 39, 88, 103, 61)(2, 6, 118, 98, 41, 84, 63, 58, 83, 5, 111, 29)(3, 109, 81, 106, 114, 67, 119, 101, 66, 12, 107, 11)(4, 116, 108, 62, 20, 55, 90, 75, 79, 80, 27, 113)(7, 60, 120, 34, 57, 69, 96, 78, 85, 65, 45, 76)(8, 13, 40, 100, 52, 17, 24, 18, 64, 47, 71, 19)(9, 87, 10, 53, 44, 56, 89, 110, 115, 28, 31, 99)(14, 117, 26, 77, 70, 42, 59, 104, 21, 72, 35, 74)(15, 46, 33, 95, 82, 51, 97, 22, 23, 43, 38, 112)(16, 30, 86, 25, 49, 94, 73, 37, 54, 50, 91, 32)

(d-)q_1 ~ (1, 41)(2, 98, 24, 102)(3, 105, 25, 59)(4, 100, 10, 43)(5, 87, 27, 44)(6, 40)(7, 82, 8, 110)(9, 61, 55, 51)(11, 47, 12, 34)(13, 48, 37, 101)(14, 109, 16, 115)(15, 52, 108, 119)(17, 21, 84, 18)(19, 89, 74, 91)(20, 72, 38, 54)(22, 79, 56, 90)(23, 114, 29, 42)(26, 64, 99, 78)(28, 53, 36, 58)(30, 111, 32, 60)(31, 104, 68, 106)(33, 57, 62, 85)(35, 66, 93, 45)(39, 118, 116, 77)(46, 107, 94, 80)(49, 95, 81, 75)(50, 83, 73, 117)(65, 113, 76, 120)(67, 103, 86, 71)(69, 97, 92, 88)

rr ~ (1, 72, 29, 77, 87, 105, 78, 13)(2, 21, 99, 71, 69, 57, 20, 95)(3, 58, 92, 43, 67, 114, 65, 59)(4, 41, 52, 102)(5, 46, 112, 56, 74, 19, 31, 118)(6, 33, 83, 11, 12, 25, 70, 111)(7, 76, 117, 50, 32, 108, 116, 36)(8, 35, 61, 37, 73, 89, 38, 68)(9, 104, 85, 48, 63, 120, 60, 93)(10, 26, 45, 106, 53, 79, 27, 80)(14, 44, 81, 18, 30, 82, 62, 91)(16, 54, 103, 40, 96, 47, 97, 51)(17, 98, 28, 55, 34, 84, 113, 75)(22, 64, 109, 42, 86, 24, 90, 115)(23, 110, 94, 101, 39, 100, 88, 119)(49, 66)

r_3 ~ (1, 63, 40, 70)(2, 36, 102, 23)(3, 19, 27, 91)(4, 17, 106, 77)(5, 89, 25, 74)(6, 96, 41, 112)(7, 53, 82, 114)(8, 58, 110, 42)(9, 115, 101, 60)(10, 84, 104, 118)(11, 88, 107, 72)(12, 97, 80, 54)(13, 119, 51, 26)(14, 93, 32, 56)(15, 90, 78, 66)(16, 35, 30, 22)(18, 69, 39, 20)(21,92, 116, 38)(24, 28, 98, 29)(31, 94, 43, 34)(33, 44, 57, 59)(37, 52, 61, 99)(45, 108, 79, 64)(46, 100, 47, 68)(48, 111, 55, 109)(49, 67, 113, 83)(50, 75, 103, 76)(62, 87, 85, 105)(65, 73, 95, 71)(81, 86, 120, 117)

r_{13} ~ (3, 7, 20, 8, 4, 10, 12, 5)(6, 17, 29, 13, 9, 19, 28, 15)(11, 18, 27, 38, 21, 24, 41, 25)(14, 32, 64, 33, 23, 47, 56, 30)(16, 37, 43, 40, 22, 45, 36, 34)(26, 48, 53, 49, 42, 39, 76, 51)(31, 61, 102, 57, 46, 63, 99, 59)(35, 70, 79, 65, 44, 73, 71, 68)(50, 90, 60, 107, 77, 116, 82, 89)(52, 95, 94, 93)(54, 62, 78, 66, 55, 84, 72, 74)(58, 87, 119, 112, 83, 91, 120, 104)(67, 97, 106, 117, 81, 98, 101, 110)(69, 88, 85, 115, 80, 113, 75, 96)(86, 114, 105, 109, 92, 118, 103, 111)(100, 108)

r_{23} ~ (1, 23, 28, 120, 53, 80, 6, 45)(2, 51)(3, 109, 79, 30, 71, 88, 97, 18)(4, 60, 99, 56, 108, 40, 46, 21)(5, 119, 63, 14, 68, 50, 8, 103)(9, 33, 17, 74, 49, 84, 42, 107)(10, 112, 73, 113, 101, 93, 59, 89)(11, 66, 52, 43, 54, 91, 105, 116)(12, 26, 102, 19, 117, 13, 44, 29)(15, 65, 76, 106, 48, 100, 39, 67)(16, 34, 118, 85, 25, 69, 104, 55)(20, 24, 110, 38, 98, 96, 31, 87)(22, 95, 62, 115, 32, 47, 114, 90)(27, 83, 86, 92, 82, 72, 77, 75)(36, 58, 111, 64)(37, 57, 78, 61, 94, 81, 41, 70)

(d-)r_1 ~ (1, 92, 34, 17, 75, 23, 30, 65)(2, 64, 69, 106, 88, 3, 120, 89)(5, 15, 38, 63, 48, 119, 115, 51)(6, 50, 113, 105, 42, 68, 79, 32)(7, 44, 9, 20, 108, 112, 116, 54)(8, 109)(10, 94, 80, 85, 57, 70, 53, 62)(11, 90, 41, 35, 60, 103, 104, 24)(12, 40, 99, 22, 19, 98, 49, 76)(13, 39, 87, 77, 16, 52, 14, 55)(18, 47, 86, 31, 45, 26, 114, 78)(21, 81, 97, 73, 102, 83, 67, 111)(25, 29, 117, 93, 95, 66, 91, 56)(27, 118, 101, 61, 37, 110, 72, 82)(28, 58, 46, 36, 100, 33, 43, 107)(59, 84, 74, 71)

j ~ (1, 41, 72, 90, 19, 111, 29, 115, 83, 78, 53, 108, 85, 55, 4)(2, 100, 99, 73, 16, 38, 82, 76, 39, 27, 87, 25, 59, 46, 65)(3, 47, 52, 7, 105, 95, 71, 75, 5, 102, 30, 18, 44, 68, 92)(6, 109, 15, 48, 88, 28, 114, 106, 66, 60, 64, 40, 91, 67, 62)(8, 43, 58, 116, 63, 86, 112, 94, 23, 20, 98, 80, 37, 93, 77)(9, 51, 49, 118, 50, 107, 81, 22, 79, 97, 32, 57, 74, 119, 104)(10, 84, 35, 33, 101, 103, 45, 89, 13, 11, 54, 26, 113, 120, 14)(12, 110, 70, 36, 61, 31, 117, 69, 56, 42, 96, 24, 17, 21, 34)

j_1 ~ (1, 77, 115, 27, 56, 29, 55, 12, 111, 76)(2, 4, 69, 100, 97, 8, 102, 73, 103, 110)(3, 62, 84, 117, 120, 82, 47, 72, 5, 106)(6, 75, 78, 118, 45, 42, 13, 87, 108, 11)(7, 35, 83, 99, 85, 112, 68, 64, 38, 22)(9, 16, 15, 96, 63, 109, 119, 66, 101, 90)(10, 21, 43, 36, 91, 25, 71, 94, 19, 65)(14, 79, 41, 26, 30, 40, 48, 52, 93, 51)(17, 23, 49, 39, 34, 113, 114, 107, 58, 28)(18, 50, 59, 57, 53, 92, 46, 88, 95, 33)(20, 70, 89, 32, 86, 61, 24, 37, 31, 60)(44, 104, 116, 98, 81, 67, 80, 74, 105, 54)

j_2 ~ (1, 76, 111, 12, 55, 29, 56, 27, 115, 77)(2, 110, 103, 73, 102, 8, 97, 100, 69, 4)(3, 106, 5, 72, 47, 82, 120, 117, 84, 62)(6, 11, 108, 87, 13, 42, 45, 118, 78, 75)(7, 22, 38, 64, 68, 112, 85, 99, 83, 35)(9, 90, 101, 66, 119, 109, 63, 96, 15, 16)(10, 65, 19, 94, 71, 25, 91, 36, 43, 21)(14, 51, 93, 52, 48, 40, 30, 26, 41, 79)(17, 28, 58, 107, 114, 113, 34, 39, 49, 23)(18, 33, 95, 88, 46, 92, 53, 57, 59, 50)(20, 60, 31, 37, 24, 61, 86, 32, 89, 70)(44, 54, 105, 74, 80, 67, 81, 98, 116, 104)

j_3 ~ (1, 53, 40, 26, 79, 43, 11, 68, 27, 2, 82, 5, 54, 116, 97, 13, 60)(3, 117, 39, 81, 74, 18, 78, 6, 36, 28, 41, 30, 107, 57, 76, 12, 71)(4, 16, 25, 108, 69, 77, 50, 109, 100, 49, 35, 9, 65, 88, 119, 86, 34)(7, 98, 104, 47, 120, 61, 112, 55, 19, 29, 85, 93, 70, 45, 95, 118, 20)(8, 96, 110, 72, 103, 15, 22, 64, 32, 48, 14, 83, 31, 24, 63, 102, 44)(10, 56, 80, 23, 21, 101, 92, 113, 62, 42, 46, 111, 99, 17, 114, 89, 75)(33, 94, 90, 52, 59, 67, 38, 87, 106, 73, 105, 115, 37, 84, 91, 51, 66)

j_4 ~ (2, 5)(4, 11)(6, 43)(7, 18)(8, 12)(9, 28)(10, 38)(13, 30)(14, 22)(15, 40)(16, 78)(17, 74)(19, 33)(20, 25)(21, 27)(23, 54)(24, 41)(29, 64)(31, 60)(32, 66)(34, 37)(35, 63)(36, 72)(39, 71)(42, 88)(44, 81)(45, 62)(46, 96)(47, 108)(48, 85)(49, 89)(50, 52)(51, 93)(53, 83)(55, 100)(57, 104)(58, 82)(59, 116)(61, 87)(65, 110)(67,80)(68, 115)(69, 107)(70, 119)(73, 113)(75, 112)(76, 111)(77, 97)(79, 103)(86, 91)(94, 109)(95, 106)(98, 99)(101, 105)(102, 114)(118, 120)

cst :

```
N1:=Stabiliser(N,1);
N12:=Stabiliser(N1,2);
for g in N do if 1^g eq 2 and 2^g eq 1 then
N12:=sub<N|N12,g>; end if; end for;
for g in N do if 1^g eq 100 and 2^g eq 108 then
N12:=sub<N|N12,g>; end if; end for;
for g in N do if 1^g eq 108 and 2^g eq 100 then
N12:=sub<N|N12,g>; end if; end for;
```

```
N16:=Stabiliser(N1,6);
for g in N do if 1^g eq 1 and 6^g eq 40 then
N16:=sub<N|N16,g>; end if; end for;
N126:=Stabiliser(N1,26);
for g in N do if 1^g eq 69 and 26^g eq 55 then
N126:=sub<N|N126,g>; end if; end for;
for g in N do if 1^g eq 107 and 26^g eq 100 then
N126:=sub<N|N126,g>; end if; end for;
N1254:=Stabiliser(N12,54);
for  g in N do if  1^g eq 69 and 2^g eq 49 and 54^g eq 13
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq 91 and 2^g eq 30 and 54^g eq 19
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq 99 and 2^g eq 34 and 54^g eq 39
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq 103 and 2^g eq 59 and 54^g eq 50
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq  118 and 2^g eq 6 and 54^g eq 55
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq 20 and 2^g eq 15 and 54^g eq 72
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq 17 and 2^g eq 4 and 54^g eq 78
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq 106 and 2^g eq 76 and 54^g eq 98
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq 16 and 2^g eq 10 and 54^g eq 104
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq 51 and 2^g eq 12 and 54^g eq 113
then N1254:=sub<N|N1254,g>; end if; end for;
for  g in N do if  1^g eq 65 and 2^g eq 5 and 54^g eq 115
then N1254:=sub<N|N1254,g>; end if; end for;
N161:=Stabiliser(N16,1);
for  g in N do if  1^g eq 6 and 6^g eq 1 and 1^g eq 6 then
N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 40 and 6^g eq 1 and 1^g eq 40
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 41 and 6^g eq 40 and 1^g eq 41
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 49 and 6^g eq 113 and 1^g eq 49
then N161:=sub<N|N161,g>; end if; end for;
for g in N do if 1^g eq 50 and 6^g eq 71 and 1^g eq 50 then
N161:=sub<N|N161,g>; end if; end for;
for g in N do if  1^g eq 63 and 6^g eq 70 and 1^g eq 63
then N161:=sub<N|N161,g>; end if; end for;
```

```
for  g in N do if  1^g eq 65 and 6^g eq 75 and 1^g eq 65
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 67 and 6^g eq 83 and 1^g eq 67
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 70 and 6^g eq 63 and 1^g eq 70
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 71 and 6^g eq 50 and 1^g eq 71
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 73 and 6^g eq 71 and 1^g eq 71
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 75 and 6^g eq 65 and 1^g eq 75
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 76 and 6^g eq 75 and 1^g eq 76
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 81 and 6^g eq 113 and 1^g eq 81
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 83 and 6^g eq 67 and 1^g eq 83
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 86 and 6^g eq 83 and 1^g eq 86
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 95 and 6^g eq 65 and 1^g eq 95
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 96 and 6^g eq 63 and 1^g eq 96
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 103 and 6^g eq 50 and 1^g eq 103
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 112 and 6^g eq 70 and 1^g eq 112
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 113 and 6^g eq 49 and 1^g eq 113
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 117 and 6^g eq 67 and 1^g eq 117
then N161:=sub<N|N161,g>; end if; end for;
for  g in N do if  1^g eq 120 and 6^g eq 49 and 1^g eq 120
then N161:=sub<N|N161,g>; end if; end for;

Order (N);
16320
Order (N1);
136
Order (N12);
32
Order (N16);
8
Order (N126);
6
```

```
Order (N1254);
24
Order (N161);
192

Generators (N1);
<(2, 3, 5, 11, 12, 27, 25, 24, 10, 7, 18, 38, 41, 20, 21,
8, 4)(6, 14, 28, 54, 64, 100, 56, 55, 29, 23, 9, 22, 43,
78, 72, 36, 16)(13, 19, 40, 74, 37, 32, 62, 108, 84, 47,
45, 66, 34, 17, 15, 33, 30)(26, 50, 88, 97, 73, 99, 119,
114, 118, 120, 102, 70, 98, 113, 77, 42, 52)(31, 58, 96,
53, 68, 111, 110, 94, 117, 109, 65, 76, 115, 83, 46, 82,
60)(35, 67, 81, 44, 80, 63, 107, 87, 49, 86, 93, 92, 51,
91, 89, 61, 69)(39, 48, 85, 71, 112, 103, 57, 101, 116, 95,
90, 106, 59, 105, 104, 79, 75),
    (3, 5, 12, 10, 4, 8, 20, 7)(6, 15, 28, 19, 9, 13, 29,
17)(11, 25, 41, 24, 21, 38, 27, 18)(14, 30, 56, 47, 23, 33,
64, 32)(16, 34, 36, 45, 22, 40, 43, 37)(26, 51, 76, 39, 42,
49, 53, 48)(31, 59, 99, 63, 46, 57, 102, 61)(35, 68, 71,
73, 44, 65, 79, 70)(50, 89, 82, 116, 77, 107, 60, 90)(52,
93, 94, 95)(54, 74, 72, 84, 55, 66, 78, 62)(58, 104, 120,
91, 83, 112, 119, 87)(67, 110, 101, 98, 81, 117, 106,
97)(69, 96, 75, 113, 80, 115, 85, 88)(86, 111, 103, 118,
92, 109, 105, 114)(100, 108)>

Generators (N12);
<(1, 2)(3, 6)(4, 9)(5, 13)(7, 19)(8, 15)(10, 17)(11,
26)(12, 28)(14, 31)(16, 35)(18, 39)(20, 29)(21, 42)(22,
44)(23, 46)(24, 48)(25, 49)(27, 53)(30, 57)(32, 63)(33,
59)(34, 65)(36, 71)(37, 73)(38, 51)(40, 68)(41, 76)(43,
79)(45, 70)(47, 61)(50, 77)(52, 94)(54, 97)(55, 98)(56,
99)(60, 82)(62, 101)(64, 102)(66, 67)(69, 109)(72, 110)(74,
81)(75, 114)(78, 117)(80, 111)(84, 106)(85, 118)(86,
88)(87, 91)(92, 113)(93, 95)(96, 103)(100, 108)(104,
112)(105, 115),
    (1, 100, 2, 108)(3, 115, 29, 92, 4, 96, 28, 86)(5,
114, 19, 69, 8, 118, 17, 80)(6, 103, 20, 88, 9, 105, 12,
113)(7, 111, 15, 75, 10, 109, 13, 85)(11, 39, 53, 38, 21,
48, 76, 25)(14, 43, 102, 44, 23, 36, 99, 35)(16, 46, 79,
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(1, 95, 117, 40, 76, 67)(2, 15, 21, 98, 64, 39)(3, 72, 37, 5, 97, 51)(4, 13, 59, 104, 61, 87)(6, 65, 86, 41, 75, 83)(7, 109, 116, 110, 60, 18)(8, 78, 94, 82, 108, 47)(9, 114, 100, 48, 58, 31)(10, 93, 27, 106, 22, 25)(11, 74, 14, 80, 91, 30)(12, 85, 26, 107, 33, 52)(16, 118, 62, 32, 17, 57)(19, 99, 77, 89, 119, 84)(20, 66, 53, 92, 79, 42)(23, 68, 90, 28, 43, 45)(24, 111, 34, 102, 115, 46)(29, 69, 55, 36, 38, 101)(35, 105, 88, 56, 44, 54)(49, 71, 63, 120, 50, 70)(73, 96, 81, 103, 112, 113),

(1, 103, 63, 117, 41, 71, 112, 83)(2, 101, 89, 37, 110, 66, 62, 35)(3, 118, 36, 116, 87, 80, 114, 46)(4, 108, 97, 109, 54, 60, 106, 78)(5, 12, 58, 21, 59, 84, 29, 47)(6, 50, 70, 67, 40, 73, 96, 86)(7, 9, 19, 22, 98, 90, 57, 61)(8, 79, 74, 51, 102, 55, 85, 56)(10, 111, 88, 64, 72, 15, 104, 115)(11, 23, 18, 27, 17, 53, 34, 105)(13, 24, 45, 91, 93, 82, 48, 33)(14, 100, 119, 68, 99, 20, 30, 69)(16, 38, 52, 92, 26, 43, 32, 31)(25, 107, 28, 94, 44, 77, 42, 39)(65, 75, 76, 95)(113, 120),

(2, 24)(3, 25)(4, 10)(5, 27)(7, 8)(9, 55)(11, 12)(13, 37)(14, 16)(15, 108)(17, 84)(18, 21)(19, 74)(20, 38)(22, 56)(23, 29)(26, 99)(28, 36)(30, 32)(31, 68)(33, 62)(34, 47)(35, 93)(39, 116)(42, 114)(43, 100)(44, 87)(45, 66)(46, 94)(48, 101)(49, 81)(50, 73)(51, 61)(52, 119)(53, 58)(54, 72)(57, 85)(59, 105)(60, 111)(64, 78)(65, 76)(67, 86)(69, 92)(71, 103)(75, 95)(77, 118)(79, 90)(80, 107)(82, 110)(83, 117)(88, 97)(89, 91)(98, 102)(104, 106)(109, 115)(113, 120),

(1, 96, 40, 112)(2, 58, 98, 114)(3, 62, 5, 57)(4, 80, 104, 11)(6, 63, 41, 70)(7, 28, 110, 23)(8, 36, 82, 29)(9, 15, 48, 64)(10, 107, 106, 12)(13, 14, 61, 30)(16, 51, 32, 37)(17, 72, 118, 97)(18, 43, 116, 68)(19, 44, 89, 105)(20, 46, 92, 34)(21, 100, 39, 31)(22, 52, 93, 26)(24, 53, 102, 42)(25, 33, 27, 85)(35, 99, 56, 119)(38, 94, 69, 47)(45, 109, 90, 60)(49, 83, 120, 86)(50, 65, 71, 75)(54, 77, 88, 84)(55, 108, 101, 78)(59, 74, 87, 91)(66, 115, 79, 111)(67, 81, 117, 113)(73, 76, 103, 95)>

Compute xx, yy, zz, and tt:

```
N:=sub<j3|x,y,z>;
T:=Transversal(j3,sub<j3|IN>);
Seq:=[i : i in [1..6156]];
Seqx:=Seq; Seqy:=Seq; Seqz:=Seq; Seqt:=Seq;

for i in [1..6156] do for j in [1..6156] do for k in
[1..6156] do for l in [1..6156] do if IN*(T[i]^x) eq
IN*T[j] then Seqx[i]:=j and IN*(T[k]^y) eq IN*T[j] then
Seqy[k]:=j and IN*(T[l]^z) eq IN*T[j] then Seqz[l]:=j;
break; break; break; end if; end for; end for; end for; end
for;

for i in [1..6156] do for j in [1..6156]  do if IN*(T[i]*t)
eq IN*T[j] then Seqt[i]:=j; break; end if; end for; end
for;

J3:=Sym(6156);
xx:=J3!Seqx; yy:=J3!Seqy; zz:=J3!Seqz; tt:=Seqt;
IN:=sub<J3|xx,yy,zz>; J3:=sub<J3|xx,yy,zz,tt>;
a:=xx; b:=yy; c:=zz;

ts:=[Id(J3):i in [1..120]];
ts[1]:=tt;
ts[2]:=tt^c;
ts[3]:=ts[2]^(a);
ts[4]:=ts[2]^(a^16);
ts[5]:=ts[2]^(a^2);
ts[6]:=ts[3]^(c);
ts[7]:=ts[2]^(a^9);
ts[8]:=ts[2]^(a^15);
ts[9]:=ts[4]^(c);
ts[10]:=ts[2]^(a^8);
ts[11]:=ts[2]^(a^3);
ts[12]:=ts[2]^(a^4);
ts[13]:=ts[5]^(c);
ts[14]:=ts[6]^(a);
ts[15]:=ts[6]^(b);
ts[16]:=ts[6]^(a^16);
ts[17]:=ts[10]^(c);
ts[18]:=ts[2]^(a^10);
ts[19]:=ts[7]^(c);
ts[20]:=ts[2]^(a^13);
ts[21]:=ts[2]^(a^14);
ts[22]:=ts[9]^a;
ts[23]:=ts[9]^(a^16);
ts[24]:=ts[2]^(a^7);
ts[25]:=ts[2]^(a^6);
ts[26]:=ts[11]^(c);
ts[27]:=ts[2]^(a^5);
ts[28]:=ts[15]^(b);
ts[29]:=ts[20]^(c);
ts[30]:=ts[14]^(b);
ts[31]:=ts[14]^(c);
ts[32]:=ts[13]^(a^5);
ts[33]:=ts[14]^(b^5);
ts[34]:=ts[16]^(b);
ts[35]:=ts[16]^(c);
ts[36]:=ts[16]^(b^2);
ts[37]:=ts[13]^(a^4);
ts[38]:=ts[2]^(a^11);
ts[39]:=ts[26]^(b^3);
ts[40]:=ts[13]^(a^2);
```

```
ts[41]:=ts[2]^(a^12);
ts[42]:=ts[21]^c;
ts[43]:=ts[6]^(a^12);
ts[44]:=ts[22]^(c);
ts[45]:=ts[13]^(a^10);
ts[46]:=ts[23]^(c);
ts[47]:=ts[13]^(a^9);
ts[48]:=ts[24]^(c);
ts[49]:=ts[25]^(c);
ts[50]:=ts[26]^(a);
ts[51]:=ts[26]^(b);
ts[52]:=ts[26]^(a^16);
ts[53]:=ts[26]^(b^6);
ts[54]:=ts[6]^(a^3);
ts[55]:=ts[6]^(a^7);
ts[56]:=ts[6]^(a^6);
ts[57]:=ts[30]^(c);
ts[58]:=ts[31]^(a);
ts[59]:=ts[33]^(c);
ts[60]:=ts[31]^(a^16);
ts[61]:=ts[47]^c;
ts[62]:=ts[32]^a;
ts[63]:=ts[32]^(c);
ts[64]:=ts[54]^(a);
ts[65]:=ts[34]^(c);
ts[66]:=ts[45]^(a);
ts[67]:=ts[35]^(a);
ts[68]:=ts[40]^(c);
ts[69]:=ts[61]^(a);
ts[70]:=ts[35]^(b^7);
ts[71]:=ts[36]^c;
ts[72]:=ts[54]^(b^2);
ts[73]:=ts[71]^(b);
ts[74]:=ts[40]^(a);
ts[75]:=ts[39]^(a^16);
ts[76]:=ts[65]^(a);
ts[77]:=ts[50]^(c);
ts[78]:=ts[43]^(a);
ts[79]:=ts[43]^(c);
ts[80]:=ts[44]^(a);
ts[81]:=ts[74]^c;
ts[82]:=ts[60]^c;
ts[83]:=ts[46]^(a^16);
ts[84]:=ts[47]^(a^16);
ts[85]:=ts[48]^(a);
ts[86]:=ts[49]^(a);
ts[87]:=ts[49]^(a^16);
ts[88]:=ts[50]^(a);
ts[89]:=ts[50]^(b);
ts[90]:=ts[60]^(b);
ts[91]:=ts[51]^a;
ts[92]:=ts[51]^(a^16);
ts[93]:=ts[92]^(a^16);
ts[94]:=ts[93]^(b);
ts[95]:=ts[94]^(b);
ts[96]:=ts[69]^(b);
ts[97]:=ts[54]^(c);
ts[98]:=ts[55]^(c);
ts[99]:=ts[56]^(c);
ts[100]:=ts[64]^(a);
ts[101]:=ts[57]^(a);
ts[102]:=ts[64]^(c);
ts[103]:=ts[96]^(c);
ts[104]:=ts[58]^(b);
ts[105]:=ts[59]^(a);
ts[106]:=ts[84]^(c);
ts[107]:=ts[63]^(a);
ts[108]:=ts[100]^(b);
ts[109]:=ts[69]^(c);
ts[110]:=ts[67]^(b);
ts[111]:=ts[86]^(b);
ts[112]:=ts[71]^(a);
ts[113]:=ts[98]^(a);
ts[114]:=ts[105]^(b);
ts[115]:=ts[105]^(c);
ts[116]:=ts[101]^(a);
ts[117]:=ts[94]^(a);
ts[118]:=ts[103]^(b);
ts[119]:=ts[112]^(b);
ts[120]:=ts[118]^(a);
```

```
(xx^3*yy^5*zz)^5*ts[70]*ts[23]*ts[51]*ts[2]*ts[1] = e
```
is a given relation.

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